

Quick Study® ACADEMIC

DYNAMICS

NOTATIONS

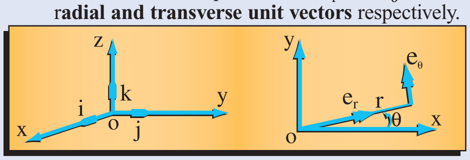
A. All **scalar quantities** are denoted by normal print, e.g., time, t. Also, the **magnitude** of vector quantities is denoted by normal print, *e.g.,* velocity magnitude, $v = |\boldsymbol{v}|$.

B. All **vectors quantities** are denoted by italic bold print, *e.g.,* velocity vector, $\boldsymbol{v}$; acceleration vector, $\boldsymbol{a}$.

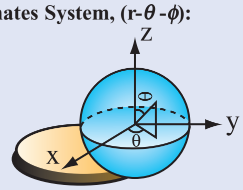
DEFINITIONS

A. **Vector Components.** Any vector can be decomposed to a certain number of components based on the **reference coordinate system**.

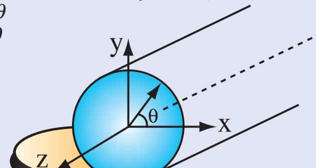
1. **Cartesian (Orthogonal) Coordinate System, x-y-z:** $\boldsymbol{a} = \boldsymbol{a}_x + \boldsymbol{a}_y + \boldsymbol{a}_z = a_x\boldsymbol{i} + a_y\boldsymbol{j} + a_z\boldsymbol{k} = \boldsymbol{a}(a_x, a_y, a_z)$, where $\boldsymbol{a}_x$, $\boldsymbol{a}_y$, $\boldsymbol{a}_z$ are the vector components and $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are the **unit vectors** along the axes x, y, z respectively.
2. **Polar Coordinate System, r-θ:** $\boldsymbol{a} = \boldsymbol{a}_r + \boldsymbol{a}_\theta = a_r\boldsymbol{e}_r + a_\theta\,\boldsymbol{e}_\theta = \boldsymbol{a}(a_r, a_\theta)$, where $\boldsymbol{a}_r$, $\boldsymbol{a}_\theta$ are the vector components and $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ are the **radial and transverse unit vectors** respectively.



3. **Spherical Coordinates System, (r-θ -φ):**
$a_x = a\cos\theta\sin\phi$
$a_y = a\sin\theta\sin\phi$
$a_z = a\cos\phi$



4. **Cylindrical Coordinate System, (r-θ-z):**
$a_x = a\cos\theta$
$a_y = a\sin\theta$
$z = z$



B. **Scalar product** between two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is defined as:
$a^o b = |\boldsymbol{a}||\boldsymbol{b}|\cos\phi = ab\cos\phi$ where ϕ is the angle formed by the two vectors.

C. **Vector product** between two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is defined as:
$c = \boldsymbol{a} \times \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\sin\phi\boldsymbol{e}_p = ab\sin\phi\boldsymbol{e}_p$, where ϕ is the angle formed by the two vectors, and $\boldsymbol{e}_p$ is the unit vector perpendicular to the plane defined by the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. Using a cartesian coordinate system the vector product can be defined as:
$c = \boldsymbol{a} \times \boldsymbol{b} = (a_yb_z - b_ya_z)\boldsymbol{i} + (a_zb_x - b_za_x)\boldsymbol{j} + (a_xb_y - b_xa_y)\boldsymbol{k}$.

D. **Particles** are hypothetical bodies that do not possess any rotational characteristics. All points of a particle have the same displacement, velocity, and acceleration.

E. **Rigid bodies** are objects whose points may have different displacement, velocity, and acceleration.

F. **Kinematics** is the study of motion without considering the forces that cause the motion. Kinematics involve displacement, velocity, acceleration, and time.

G. **Kinetics** is the study of motion as related to the forces causing the motion. Kinetics involve force, mass, and acceleration.

H. **Path** is the curve that a particle follows as it moves through the space. The path can be a space curve called **tortouos** or a plane curve called **plane path**.

KINEMATICS PARTICLE MOTION

RECTILINEAR MOTION

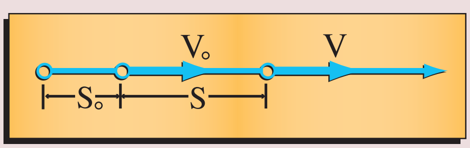
Rectilinear motion *is whenever particles move along a straight line. The governing equations regarding acceleration, a, velocity, v, and displacement (position coordinate), s, for rectilinear motion are:*

$a = dv/dt = d^2s/dt^2 = v(dv/ds)$;
$v = ds/dt = \int a\,dt$; $s = \int v dt = \int [\int a dt]dt$.

Generally, all three variables, a, v, and s, are vectors. However, in the above equations these variables are treated as scalars, since the motion is rectilinear and their direction can be defined only by their sign (positive or negative).

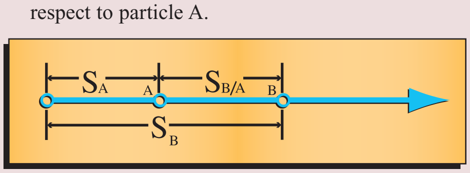
A. Formulas for Uniformly Accelerated Rectilinear Motion. Denoting the initial conditions (t = 0) of the various variables by the subscript (o), the relationships between position coordinate, s, velocity, v, acceleration, a, and time, t, are given as follows:

Given:		Estimate:
s_o, v_o, a, t	»	$s = s_o + v_ot + {}^1/_2\,at^2$
s_o, v_o, v, a	»	$s = s_o + {}^1/_2(v^2 - v_o^2)/a$
s_o, v_o, v, t	»	$s = s_o + {}^1/_2(v_o + v)t$
s_o, s, a, t	»	$v_o = (s - s_o)/t - {}^1/_2\,at$
v_o, a, t	»	$v = v_o + at$
s_o, s, v_o, a	»	$v = [v_o^2 + 2a(s - s_o)]^{1/2}$
v_o, v, t	»	$a = (v - v_o)/t$
s_o, s, v_o, t	»	$a = 2[(s - s_o) - v_ot]/t^2$
s_o, s, v_o, v	»	$a = {}^1/_2(v^2 - v_o^2)/(s - s_o)$
v_o, v, a	»	$t = (v - v_o)/a$
s_o, s, v_o, a	»	$t = \{[2a(s - s_o) + v_o^2]^{1/2} - v_o\}/a$
s_o, s, v_o, v	»	$t = 2(s - s_o)/(v_o + v)$



B. **A main example of uniformly accelerated rectilinear motion of free falling particles.**
Other types of rectilinear motion include:
Non-accelerating motions, $s = s_o + v_ot$; $v = v_o$; $a = 0$, or motions where the displacement decreases exponentially in time at a constant rate, -k, $s = s_o\exp(-kt)$; $v = -ks_o\exp(-kt)$; and $a = k^2s_o\exp(-kt)$.

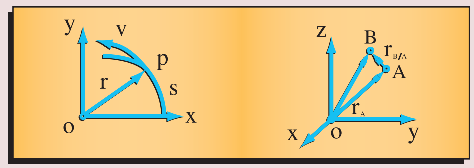
C. **Relative Motion Between Two Particles A and B.**
The position coordinate, velocity, and acceleration of particle B as related to particle A are defined as:
$s_B = s_A + s_{B/A}$; $v_B = v_A + v_{B/A}$; $a_B = a_A + a_{B/A}$, where $s_{B/A}$ is the distance between the two particles A and B, and $v_{B/A}$, $a_{B/A}$ are respectively the relative velocity and acceleration of particle B with respect to particle A.



CURVILINEAR MOTION

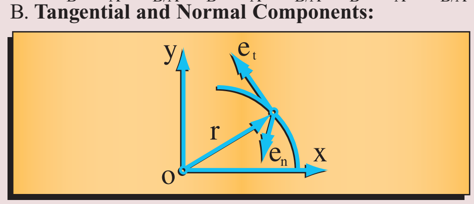
Curvilinear motion *is the motion where particles move along a curved path. The position of a particle is given by the position vector* $\boldsymbol{r}$*. The velocity,* $\boldsymbol{v}$*, and the acceleration,* $\boldsymbol{a}$*, for curvilinear motion are defined as:*

$\boldsymbol{v} = d\boldsymbol{r}/dt$, $|\boldsymbol{v}| = v = ds/dt$ (particle velocity, $\boldsymbol{v}$, is tangent to the particle's path, s), $\boldsymbol{a} = d\boldsymbol{v}/dt$ (particle acceleration, $\boldsymbol{a}$, is not tangent to the particle's path, s).



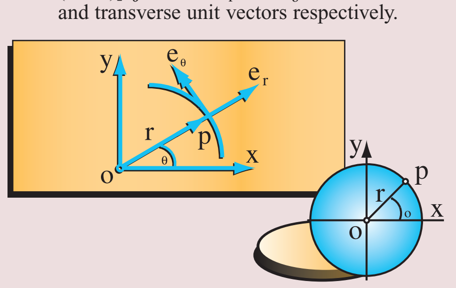
A. **Cartesian Coordinate System (Rectangular Components):**
$\boldsymbol{r} = \boldsymbol{r}(x, y, z)$; $\boldsymbol{v} = \boldsymbol{v}(v_x, v_y, v_z)$; $\boldsymbol{a} = \boldsymbol{a}(a_x, a_y, a_z)$;
$v_x = dx/dt$; $v_y = dy/dt$; $v_z = dz/dt$,
$a_x = dv_x/dt$; $a_y = dv_y/dt$; $a_z = dv_z/dt$.
Circular Periodic Motion:
$x = r\cos(\theta t)$; $y = r\sin(\theta t)$; $x^2 = y^2 = r^2$;
$v_x = -r\theta\sin(\theta t)$; $v_y = r\theta\cos(\theta t)$; $v = r\theta$,
$a_x = -r\theta^2\cos(\theta t)$; $a_y = -r\theta^2\sin(\theta t)$; $a = r\theta^2$
Relative Motion Between Two Particles A and B:
$\boldsymbol{r}_B = \boldsymbol{r}_A + \boldsymbol{r}_{B/A}$; $\boldsymbol{v}_B = \boldsymbol{v}_A + \boldsymbol{v}_{B/A}$; $\boldsymbol{a}_B = \boldsymbol{a}_A + \boldsymbol{a}_{B/A}$

B. **Tangential and Normal Components:**



1. **Two-Dimensional Motion:** For plane motion, the particle acceleration can be separated into two components, one tangential, $\boldsymbol{a}_t$, and one normal, $\boldsymbol{a}_n$: $\boldsymbol{v} = v\boldsymbol{e}_t$; $\boldsymbol{a} = \boldsymbol{a}_t + \boldsymbol{a}_n = (dv/dt)\,\boldsymbol{e}_t + (u^2/\rho)\boldsymbol{e}_n$. where $\boldsymbol{a}_t$ is the tangential acceleration, $\boldsymbol{a}_n$ is the normal acceleration, ρ is the radius of curvature, $\boldsymbol{e}_t$ is the tangential unit vector, and $\boldsymbol{e}_n$ is the normal unit vector to the curved path of the particle.
2. **Three-Dimensional Motion:** Osculating plane in a three dimensional motion is the plane in the neighborhood of the moving particle that includes the unit vectors, $\boldsymbol{e}_t$, (tangent) and $\boldsymbol{e}_n$, (principal normal). The **binormal unit vector**, $\boldsymbol{e}_b$, is perpendicular to the osculating plane.

C. **Polar Coordinates System (Radial and Transverse Components in Plane Motion):**
1. **Particle Velocity:**
$\boldsymbol{v} = v_r\boldsymbol{e}_r + v_\theta\,\boldsymbol{e}_\theta = (dr/dt)\boldsymbol{e}_r + r(d\theta/dt)\boldsymbol{e}_\theta$;
2. **Particle Acceleration:**
$\boldsymbol{a} = a_r\boldsymbol{e}_r + a_\theta\,\boldsymbol{e}_\theta = [d^2r/dt^2 - r(d\theta/dt^2]\boldsymbol{e}_r + [r(d^2\theta/dt^2) + 2(dr/dt)(d\theta/dt)]\boldsymbol{e}_\theta$, where $\boldsymbol{e}_r$ and $\boldsymbol{e}_\vartheta$ are the unit radial and transverse unit vectors respectively.



CIRCULAR MOTION

***Circular motion** is whenever the particle moves on a circular path. The governing equations for the angular acceleration, a, angular velocity, w, and angular position, q, in circular motion are:*

$a = d\omega/dt = d^2\theta/dt^2$; $\omega = d\theta/dt = \int \alpha dt$;
$\theta = \int \omega dt = \int [\int \alpha dt]dt$.

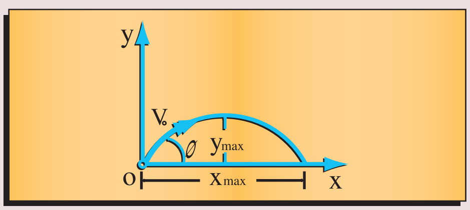
A. **Relationships Between Rectangular and Polar Components For Circular Motion:**
1. **Tangential Velocity: (v_t):** $v_t = \omega r$; $v_{tx} = -v_t \sin\theta$; $v_{ty} = v_t \cos\theta$, where θ is between the velocity vector and the horizontal axis, x.
2. **Tangential Acceleration (a_t):** $a_t = \alpha r$, where α is the magnitude of the angular acceleration.
3. **Normal Acceleration (a_n):** $a_n = v_t^2/r = r\omega^2 = v_t\omega$.
4. **Coriolis Acceleration (a_c):** $a_c = 2v_t\omega$.

PROJECTILE MOTION ON AN X-Y PLANE

A. **Trajectory Coordinates:**
$x = (v_o\cos\phi)t$; $y = v_o\sin\phi t - {}^1/_2 gt^2$;
$y = v_o\sin\phi x/(v_o\cos\phi) - {}^1/_2\, g\, [x/(v_o\cos\phi)]^2$;
The equation is a **parabola**.
1. **Maximum Horizontal Distance (Range):**
$x_{max} = (v_o^2\sin 2\phi)/g$;
For $\phi = 45°$ the range is maximum.
2. **Maximum Height:**
$y_{max} = ({}^1/_2 v_o^2\sin 2\phi)/g$, where v_o is the initial velocity, and ϕ is the angle between the initial velocity vector and the horizontal axis, x.

B. **ProjectileVelocity:**
$v = (v_o^2 - 2gy)$; $v_x = v_o\cos\phi$; $v_y = v_o\sin\phi - gt$.

C. **Total Flight Time:**
$t_{total} = 2v_o\sin\phi/g$.



WORK, ENERGY, AND POWER

A. **Work, W,** performed by a force in the direction of motion (positive work) is estimated as:
1. **Variable Force:** $W = \int \boldsymbol{F} \circ ds$
2. **Constant Force:** $W = \boldsymbol{F} \circ \int ds$
3. **Variable Torque:** $W = \int \boldsymbol{T} \circ d\theta$
4. **Constant Torque:** $W = \boldsymbol{T} \circ \int d\theta$
If $\boldsymbol{F} \perp s$ or $\boldsymbol{T} \perp \theta$ then $W = 0$.

B. **Energy, E,** is defined as the capacity to perform work.

C. **Work-Energy Principle.** In classical mechanics, energy cannot be created or destroyed but it can be transformed from one type of energy to another. Thus, in a conservative system, the energy level of the system increases whenever there is positive work performed: dE = W.
1. **Potential Energy, (P.E.):**
   a. **P.E. from Particle Weight:** $E_p = mgh$.
   b. **P.E. from Gravitational Force:** $E_p = -GmM/r$.
   c. **P.E. from Elastic Spring:** $E_p = {}^1/_2 kx^2$, where k is the spring constant.
2. **Kinetic Energy (K.E.):**
   a. **Rectilinear Motion:** $E_k = mv^2/2$;
   b. **Circular Motion:** $E_k = Iw^2/2$.

D. **Power, P,** is the amount of work performed per unit time: $P = W/dt$; $P = Fv$; $P = T\omega$.

E. **Mechanical efficiency** is the ratio of power output over power input.

KINETICS PARTICLE MOTION

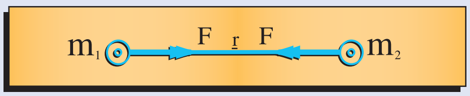
NEWTON'S LAWS

A. **First Law (Inertia Law):** A particle will remain at rest or continue to move with constant velocity, unless an external unbalanced force acts on the particle.

B. **Second Law:** The acceleration, $\boldsymbol{a}$, of a particle of mass, m, is directly proportional to the resultant force, $\Sigma\boldsymbol{F}$, acting on the particle and inversely proportional to the mass of the particle. The resulting acceleration has the same direction with the force: $\boldsymbol{a} = \Sigma\boldsymbol{F}/m$.
1. **Cartesian Acceleration Components:**
$a_x = \Sigma\boldsymbol{F}_x/m$; $a_y = \Sigma\boldsymbol{F}_y/m$; $a_z = \Sigma\boldsymbol{F}_z/m$.
2. **Tangential and Normal Force Components:**
$\Sigma\boldsymbol{F}_t = m(dv/dt)$; $\Sigma\boldsymbol{F}_n = m(v^2/r)$.
3. **Dynamic Equilibrium:** $\Sigma\boldsymbol{F} - m\boldsymbol{a} = 0$.

C. **Third Law:** For every acting force there is a reacting force of equal strength and opposite direction: $\boldsymbol{F}_{reacting} = -\boldsymbol{F}_{acting}$.

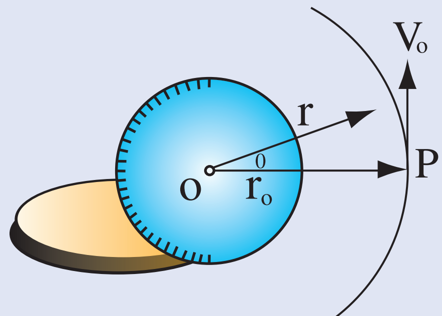
D. **Universal Gravitation Law:** The attractive force between two bodies is proportional to the product of their masses, m_1, m_2, and inversely proportional to the squarepower of the distance, $\boldsymbol{r}$, between their centroids:
$\boldsymbol{F} = Gm_1m_2/\boldsymbol{r}^2$.
The proportionality constant equals:
$G = (66.73 \pm 0.03)\times10^{-12} m^3/kg\text{-}s^2 (3.44\times10^{-8} ft^4/lbf\text{-}sec^4)$.



1. **Gravitational Force of the Earth:**
$\boldsymbol{F} = m\boldsymbol{g} = mg\boldsymbol{k}$, where the gravitational acceleration, g, is approximately equal to:
$g = 9.81 m/s^2$ ($32.2\ ft/s^2$), and $\boldsymbol{k}$ is the unit vector pointing at the center of the earth. The acceleration due to gravity at a specific location, can be estimated more accurately by using the formula:
$g = 32.089(1 + 0.00524 \sin^2\phi)(1 - 0.000000096h)$ [ft/s²], where ϕ is the latitude and h is the elevation in feet.
2. **Orbital Trajectory Subject to a Gravitational Force:**
The **governing equation of the conic with eccentricity** $e = Ch^2/GM$, describing the trajectory of a particle subject to gravitational force is given as: $(1/r) = GM/h^2 + C\cos\theta$; where r is the magnitude of the position vector, θ is the polar angle, M is the mass of the attracting body (e.g., earth), and C, h are constants. Under the initial conditions (t = 0):
$v = v_o$(where $v_o \perp r_o$), $r = r_o$, $\theta = 0$ the constant h is estimated as: $h = r_o v_o$.

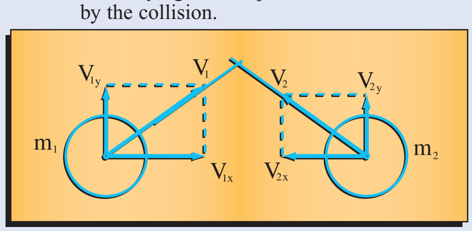
For $\left[\begin{array}{l} e < 1 \\ e = 1 \\ e > 1 \end{array}\right.$ the conic is a/an $\left[\begin{array}{l} \text{ellipse,} \\ \text{parabola,} \\ \text{hyperbola.} \end{array}\right.$

3. Escape velocity, v_{esc}, is the minimum initial velocity required to allow the particle to escape from and never return to its starting position: $v_{esc} = (2GM/r_o)^{1/2}$

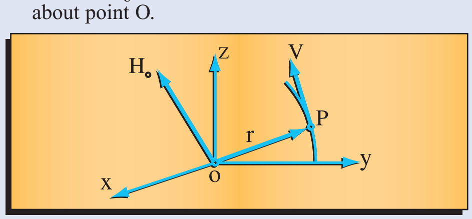


IMPULSE AND MOMENTUM

A. **Linear momentum, $\boldsymbol{L}$,** is defined as: $\boldsymbol{L} = mv$; $\Sigma\boldsymbol{F} = d\boldsymbol{L}/dt$; and; $\Sigma\boldsymbol{F}\, dt = d\boldsymbol{L} = mdv$.
1. **Linear Impulse, $\boldsymbol{Imp}_{1\text{-}2}$,** is defined as:
$\boldsymbol{Imp}_{1\text{-}2} = \Sigma\boldsymbol{F}\Delta t = m(\boldsymbol{v}_2 - \boldsymbol{v}_1)$, $\Delta t = t_2 - t_1$.
2. **Direct impact** between two particles occurs whenever the velocities of the particles are perpendicular to the tangential plane at the point of their contact. **Central impact** between two particles occurs whenever the force of impact is along the line connecting the centroids of the colliding particles. The velocities after a direct central impact of two particles of equal mass m is estimated as: $v_2' - v_1' = e(v_2 - v_1)$, where v_i and v_i' (i = 1, 2) are respectively the velocities before and after the impact. The factor e is called **coefficient of restitution** and incorporates the effects of frictional and other energy losses.
   a. **Perfect Elastic Impact:** e = 1
   b. **Inelastic (Plastic) Impact:** 0 < e < 1
   c. **Perfect Plastic Impact:** e = 0 For perfect elastic impact (e = 1), the velocities after the collision of two particles can be estimated by using along with the impact equation, either the momentum or the kinetic energy equations as:
   $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$, or
   $m_1v_1^2 + m_2v_2^2 = m_1(v_1')^2 + m_2(v_2')^2$.
   If ($e \neq 1$), then the kinetic energy equation cannot be used since there are unknown energy losses.
   d. For **oblique central impact**, the coefficient of restitution should be estimated along the line that is normal to the tangential plane at the point of contact. The velocity components laying on this plane remain unaffected by the collision.



B. **Angular momentum about point $\boldsymbol{O}$, $\boldsymbol{H}_o$,** is defined as: $\boldsymbol{H}_o = \boldsymbol{r} \times (m\boldsymbol{v})$; $\boldsymbol{H}_o = m[(yv_z - zv_y)\boldsymbol{i} + (zv_x - xv_z)\boldsymbol{j} + (xv_y - yv_x)\boldsymbol{k}]$; and; $\Sigma\boldsymbol{M}_o = d\boldsymbol{H}_o/dt$, where $\Sigma\boldsymbol{M}_o$ is the resultant moment of the forces about point O.



1. **Plane Motion-Radial and Transverse Resultant Force Components:** $\Sigma\boldsymbol{F}_r = m[(d^2r/dt^2) - r(d\theta/dt)^2]$; $\Sigma\boldsymbol{F}_\theta = m[r(d^2\theta/dt^2) + 2(dr/dt)(d\theta/dt)]$.
2. **Circular Motion:** $T\,dt = I\,d\omega$; where T is the torque and I is the moment of inertia.

KINETICS OF SYSTEM OF PARTICLES

A. **Resultant Forces and Moments About Point O for a System of N Number of Particles:**
$$\Sigma_{i=1}^{i=N}\boldsymbol{F}_i = \Sigma_{i=1}^{i=N}(m_i\boldsymbol{a}_i); \qquad \Sigma_{i=1}^{i=N}(\boldsymbol{r}_i \times \boldsymbol{F}_i) = \Sigma_{i=1}^{i=N}[\boldsymbol{r}_i \times (m_i\boldsymbol{a}_i)]$$

B. **Linear and Angular Momentum of a System of N Number of Particles:**
$$\boldsymbol{L} = \Sigma_{i=1}^{i=N}(m_i\boldsymbol{v}_i); \qquad \boldsymbol{H}_o = \Sigma_{i=1}^{i=N}[\boldsymbol{r}_i \times (m_i\boldsymbol{v}_i)]$$

C. **Mass Center of System of N Number of Particles:**

$$r_G = [\Sigma_{i=1}^{i=N}(m_i r_i)]/(\Sigma_{i=1}^{i=N} m_i)$$

D. **Moment Resultant, About the Mass Center, G, of N Number of Particles:**

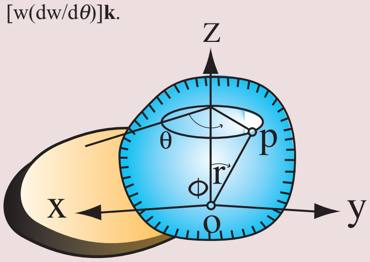
$$H_G = \Sigma_{i=1}^{i=N}[r_i' \mathbf{x} (m_i v_i)]; \qquad \Sigma_{i=1}^{i=N} M_G = dH_G/dt.$$

KINEMATICS-RIGID BODIES

A. **Translation of a rigid body** is the motion where all points of the body have the same velocity and the same acceleration at any time. If the velocity and the acceleration are not the same for all points of the body, then the motion is rotational.

B. **Rotational Motion About a Fixed Axis:**

1. **Velocity:**
   $v = dr/dt = w\mathbf{x}r$; $|v| = ds/dt = r\,(d\theta/dt)\sin\phi$.
   a. **Angular Velocity:**
   $\mathbf{w} = w\mathbf{k} = (d\theta/dt)\mathbf{k}$.
2. **Acceleration:**
   $a = a\mathbf{x}r + w\mathbf{x}(w\mathbf{x}r)$.
   a. **Angular Acceleration:**
   $\alpha = a\mathbf{k} = (d\omega/dt) = d\omega/dt\ \mathbf{k} = (d^2\theta/dt^2)\mathbf{k} = [w(dw/d\theta)]\mathbf{k}$.

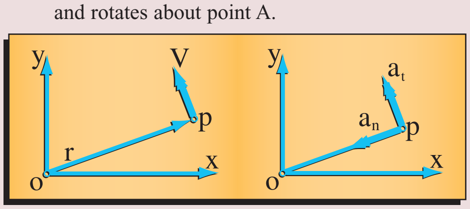


C. **Motion of a 2-D Body Located on a Plane Perpendicular to the Axis of Rotation:**

1. **Velocity:**
   $v = w\mathbf{x}r = \omega \mathbf{k} \mathbf{x} r$.
2. **Acceleration:**
   a. **Tangential:**
   $a_1 = \alpha \mathbf{x} r = \alpha \mathbf{k} \mathbf{x} r$; $a_t = \alpha r$
   b. **Normal:**
   $a_n = -\omega^2 r$; $a_n = \omega^2 r$

D. **Translation and Rotation in 2-D (Plane Motion):**

1. **Velocity:**
   $v_B = v_A + v_{B/A} = v_A + \omega\mathbf{x}r_{B/A}$; $|v_{B/A}| = \omega r$, where A and B are two points of the same rigid body that translates and rotates about point A.
2. **Acceleration:**
   $a_B = a_A + a_{B/A} = a_A + (a_{B/A})_n + (a_{B/A})_t$; $a_{B/A} = [(\omega^2 r)^2 + (\alpha r)^2]^{1/2}$, where again A and B are two points of the same rigid body, which translates and rotates about point A.

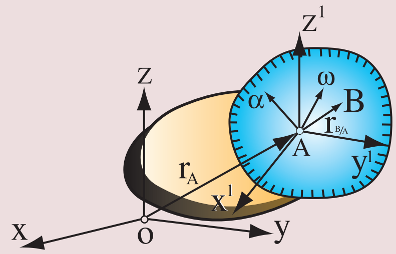


E. **Motion Relative to a Rotating Coordinate System With an Angular Velocity W:**

1. **Velocity:**
   $v_p = v_{p'} + v_{P/S} = \omega \mathbf{x} r + dr/dt|_{x'y'z'}$, where xyz is the fixed system and x'y'z'is the rotating system, v_p is the absolute velocity of point P, $v_{p'}$ is the velocity of point P' of the moving frame S (P = P'), and $v_{P/S}$ is the velocity of point P relative to the moving frame S.
2. **Acceleration:**
   $a_P = a_{P'} + a_{P/S} + a_c$; $a_c = 2\omega \mathbf{x} v_{P/S}$. Where a_P is the absolute acceleration, $a_{P'}$ is the acceleration of point P' of the moving frame S (P = P'), $a_{P/S}$ is the acceleration of P relative to the moving frame S, and a_c is the complementary (Coriolis) acceleration.

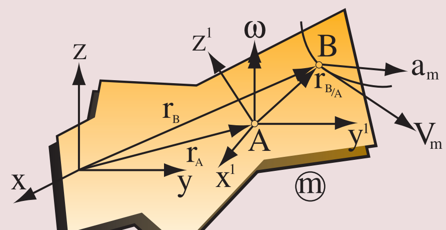
F. **Translation and Rotation in 3-D (Space) Motion:**

1. **Velocity:**
   $v_B = v_A + v_{B/A} = v_A + \omega \mathbf{x} r_{B/A}$.
2. **Acceleration:**
   $a_B = a_A + a_{B/A} = a_A + \alpha \mathbf{x} r_{B/A} + \omega\mathbf{x}(\omega\mathbf{x}r_{B/A})$.

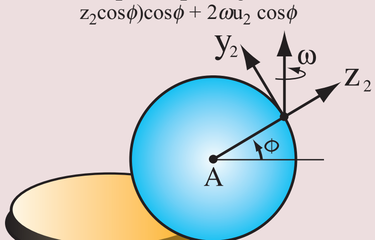


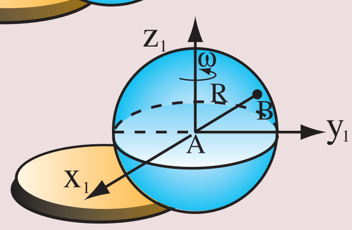
If the inertial coordinate system xyz is denoted by the subscript (i) and the moving coordinate system by the subscript (m), then $r_B = r_A + r_{B/A}$. Acceleration for a moving observer $(d_m{}^2 r_{B/A})/dt^2 = d_m[(d_m r_{B/A})/dt]/dt$. Acceleration for an inertial observer $(d_i{}^2 r_{B/A})/dt^2 = d_i[(d_i r_{B/A})/dt]/dt$.
Relationships between the intertial and the moving observer $(d_i r_B)/dt = (d_i r_A)/dt + (d_m r_{B/A})/dt = \omega \mathbf{x} r_{B/A}$ and $(d_i{}^2 r_B)/dt^2 = (d_i{}^2 r_A)/dt^2 + (d_m{}^2 r_{B/A})/dt^2 + (d_i\omega)/dt\ \mathbf{x}\ r_{B/A} + \omega\ \mathbf{x}(\omega\ \mathbf{x}\ r_{B/A}) + 2\ \omega\ \mathbf{x}\ (d_m r_{B/A})/dt$ or $(d_m{}^2 r_{B/A})/dt^2 = (d_i{}^2 r_B)/dt^2 - [(d_i{}^2 r_A)/dt^2 + (d_i\omega)/dt\ \mathbf{x}\ r_{B/A} + \omega\ \mathbf{x}\ (\omega\mathbf{x}r_{B/A}) + 2\ \omega\ \mathbf{x}\ (d_m r_{B/A})/dt]$.

The term $\omega\,\mathbf{x}\,(\omega\ \mathbf{x}\ r_{B/A})$ is the centripetal acceleration and the term $(d_i\omega)/dt$ is the angular acceleration. The above equations can be used to describe motion with reference to the sun-earth system, as experienced by an observer on earth.



3. **Using Earth as the Inertial System**
   a. **Acceleration:**
   $(d^2r_2)/dt^2 = (d^2r_1)/dt^2 - [\ \omega\ \mathbf{x}\ (\omega\ \mathbf{x}\ R) + \omega\ \mathbf{x}\ (\omega\ \mathbf{x}\ r_2) + 2\ \omega\ \mathbf{x}\ (dr_2)/dt]$
   In component form, the various terms read as:
   $\omega\ \mathbf{x}\ R = \mathbf{i}\omega R\cos\phi$
   $\omega\ \mathbf{x}\ (\omega\ \mathbf{x}\ R) = j(\omega^2 R\sin\phi\cos\phi + k(-\omega^2 R\cos^2\phi)$
   $\omega\ \mathbf{x}\ r_2 = \mathbf{i}(\omega z_2\cos\phi - \omega y_2\sin\phi) + \mathbf{j}(\omega x_2\sin\phi) + \mathbf{k}(\omega x_2\cos\phi)$
   $\omega\ \mathbf{x}\ (\omega\ \mathbf{x}\ r_2) = \mathbf{i}\omega^2 x_2(-\cos^2\phi - \sin^2\phi) + \mathbf{j}(\omega^2\sin\phi)(z^2\cos\phi - y^2\sin\phi) - \mathbf{k}\omega^2\cos\phi\ (z_2\cos\phi - y_2\sin\phi)$ $\omega\ \mathbf{x}\ (d^2r_2)/dt = \omega\ \mathbf{x}\ v_2 = \mathbf{i}\omega(\omega^2\cos\phi - v_2\sin\phi) + \mathbf{j}(\omega u_2\sin\phi) + \mathbf{k}(-\omega u_2\cos\phi)$
   b. $a_x = du_1/dt + \omega^2 x_2 + 2\omega(v_2\sin\phi - w_2\cos\phi)$ $a_y = dv_1/dt - w^2(R\cos\phi - y_2\sin\phi + z_2\cos\phi)\sin\phi - 2\omega u_2\sin\phi$ $a_z = dw_1/dt + w^2(R\cos\phi - y_2\sin\phi + z_2\cos\phi)\cos\phi + 2\omega u_2\cos\phi$





KINETICS-RIGID BODIES

A. **Fundamental Equations:**

1. **Motion of the Center of Mass, G:**
   $\Sigma F = \Sigma(m a_G)$, where a_G is the acceleration of the center of mass, **G**.
2. **Motion Relative to Centroidal Coordinate System:**
   $\Sigma M_G = dH_G/dt$, where ΣM_G is the moment about the center of mass, **G**.

B. **2-D (Plane) Motion:**

1. **Angular Momentum:**
   $H_G = I_G\omega$; $dH_G/dt = I_G(d\omega/dt) = I_G\alpha$, where I_G is the moment of inertia of the body about a centroidal axis normal to the coordinate system, and ω is the angular velocity.

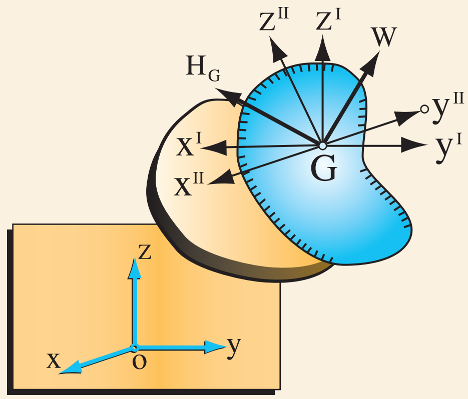
C. **3-D (Space) Motion:**

1. **Angular Momentum for Fixed Coordinate System, x'-y'-z':**

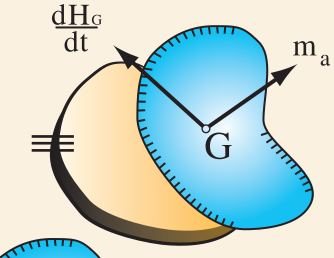
$$\begin{bmatrix} H_{x'} \\ H_{y'} \\ H_{z'} \end{bmatrix} = \begin{bmatrix} I_{x'} & -I_{x'y'} & -I_{x'z'} \\ -I_{y'x'} & I_{y'} & -I_{y'z'} \\ -I_{z'x'} & -I_{z'y'} & I_{z'} \end{bmatrix} \begin{bmatrix} w_{x'} \\ w_{y'} \\ w_{z'} \end{bmatrix}$$

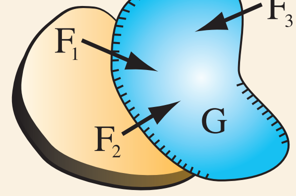
2. **Angular Momentum for Principal Axes of Inertia, x*-y*-z*:**

$$\begin{bmatrix} H_{x'} \\ H_{y'} \\ H_{z'} \end{bmatrix} = \begin{bmatrix} I_{x'} & 0 & 0 \\ 0 & I_{y'} & 0 \\ 0 & 0 & I_{z'} \end{bmatrix} \begin{bmatrix} \omega_{x'} \\ \omega_{y'} \\ \omega_{z'} \end{bmatrix}$$



3. **Angular Momentum About Point O:**
   $H_o = r_G\ \mathbf{x}\ (mv_G) + H_G$.
4. **General 3-D Motion:**
   $dH_G/dt = (dH_G/dt)_{x''y''z''} + \mathbf{\Omega}\ \mathbf{x}\ H_G$, where H_G is the angular momentum with respect to the coordinate system x'- y'- z' of fixed orientation, $(dH_G/dt)_{x''y''z''}$ is the rate of change of the angular momentum with respect to the rotating coordinate system, x"- y"- z", and $\mathbf{\Omega}$ is the angular velocity of the rotating coordinate system, x"- y"- z".
5. **Kinetic Energy Referred to the Principal Axes of Inertia x*y*z*:**
   $E_k = {}^1/_2\ mv_G{}^2 + {}^1/_2\ (I_{x*}\omega_{x*}{}^2 + I_{y*}\omega_{y*}{}^2 + I_{z*}\omega_{z*}{}^2)$.
6. **D'Alembert's Principle:**
   The system of the external forces acting on a body is equivalent to the effective forces of the body, i.e., ma and dH_G /dt.





VIBRATIONS

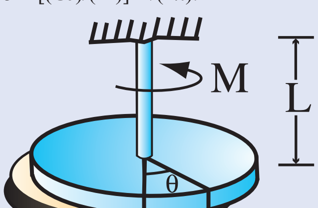
MECHANICAL VIBRATIONS

Involves the study of the motion of particles and rigid bodies, oscillating about a position of equilibrium.

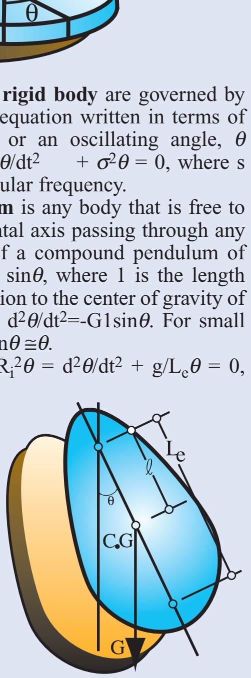
FREE VIBRATIONS

A. **Free vibrations of a particle involves** the study of the motion of a particle subject to a restoring force proportional to the displacement.

1. The **governing equation for simple harmonic motion** is: $d^2x/dt^2 + (k/m)x = d^2x/dt^2 + \sigma^2x = 0$; $\sigma^2 = k/m$, where x is the displacement, k is the spring constant, m is the mass of the oscillating particle, and σ is the **circular (natural) frequency**.
   a. **Displacement, Maximum Velocity and Maximum Acceleration:** $x(t) = x_{max}\sin(\sigma t + \phi)$; $v_{max} = x_{max}\sigma$; $\alpha_{max} = x_{max}\sigma^2$; $x_{max}=[(x_o)^2 + (v_o/\sigma)^2]^{1/2}$; $\phi = \tan^{-1}[v_o/(\sigma x_o)]$, where x_{max} is the maximum displacement, x_o and v_o are the initial displacement and velocity respectively, and ϕ is the phase angle.
   b. **Period of Vibration:** $T = (2\pi)/\sigma$.
   c. **Frequency of Vibration:** $f = 1/T= \sigma/(2\pi)$.
   d. For small oscillations of a **simple pendulum motion** the circular frequency is defined as: $\sigma = (g/l)^{1/2}$.
   e. **Tortional Vibration** of a disk, with respect to an axis perpendicular to its center, is defined as $I\ d^2\theta/dt^2 = m$, where I is the moment of inertia, θ is the angle of rotation, and M is the moment. Generally, $M = -k\theta$, where k is the tortional spring constant. For a cylindrical shaft of length L and shear modulus of elasticity G, the angle of rotation is given as $\theta = (ML)/(GJ)$, where J is the polar moment of inertia. Thus, the governing equation becomes $d^2\theta/dt^2 + (GJ)/(IL)\theta = 0$. The period of tortional vibration is $T = 2\pi[(IL)/(GJ)]^{1/2}$.
   The frequency of tortional vibration is $\sigma = [(GJ)/(IL)]^{1/2}/(2\pi)$.

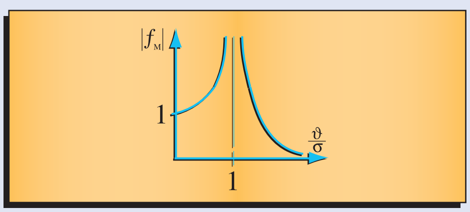


B. **Free vibrations of a rigid body** are governed by the same differential equation written in terms of the displacement, x, or an oscillating angle, θ :$d^2x/dt^2 + \sigma^2x = 0$; $d^2\theta/dt^2 + \sigma^2\theta = 0$, where s is an appropriate circular frequency.
Compound Pendulum is any body that is free to rotate about a horizontal axis passing through any point. The moment of a compound pendulum of weight G is $M = G1\sin\theta$, where 1 is the length from the point of rotation to the center of gravity of the body. Thus, $mR_i^2\ d^2\theta/dt^2 = -G1\sin\theta$. For small angle of oscillation $\sin\theta \cong \theta$.
Thus, $d^2\theta/dt^2 + (lg)/R_i^2\theta = d^2\theta/dt^2 + g/L_e\theta = 0$, where $L_e = R_i^2/1$ is the equivalent length of the compound pendulum. The point at the end of the equivalent length is called center of oscillation.



C. **Damped free vibrations** subject to **viscous damping** are described by the differential equation: $m(d^2x/dt^2) + c(dx/dt) + kx = 0$, where c is the **coefficient of viscous damping.**
1. **Critical Damping Coefficient:** $c_{cr} = 2m(k/m)^{1/2} = 2m\sigma$.
2. **Damping Ratio:** $R = c/c_{cr}$
   a. **Heavy Damping (Overdamped):** $R > 1$, (the system returns to its equlibrium position without any oscillations).
   b. **Critical Damping (Dead-beat Motion):** $R = 1$, (the system returns to its equilibrium position without any oscillations).
   c. **Light Damping (Underdamped):** $R < 1$, (the system returns to its equilibrium position after an attenuating oscillatory motion).
3. **Displacement:** $\mathbf{x}(t) = \exp(-\mu t)\ [c_1\cos(\sigma_d t) + c_2\sin(\sigma_d t)]$; $m = c/(2m)$; where c_1 and c_2 are constants, μ is the damping modulus, and σ_d is the damped frequency defined as: $\sigma_d = (\sigma^2 - \mu^2)^{1/2}$.
4. **Forced Vibrations** occur whenever a system is subject to a periodic force, P(t): $P(t) = P_0\sin(\vartheta t)$ where P_0 is the amplitude of the force, and ϑ is the forced frequency.

D. **Vibrations Without Damping.** The governing equation for the motion is given as: $m(d^2x/dt^2) + kx = P_0\sin(\vartheta t)$. The general solution is obtained by adding a particular solution x_p to the solution of the homogeneous equation. The form of the particular solution is: $x_p = x_{max}\sin(\vartheta t)$.
1. The **steady-state vibration** of the system is described by the particular solution.
2. The **transient free vibration** of the system is described by the solution of the homogeneous equation and it can be practically neglected.
3. **Magnification Factor:** $f_M = x_{max}/(P_0/k) = 1/[1-(\vartheta/\sigma)^2]$.
   The forcing is in resonance with the system if the amplitude of the forced vibration becomes theoretically infinite ($f_M \rightarrow \infty$), i.e., whenever the forced frequency, ϑ, equals the natural frequency, σ.



E. **Damped Vibrations.** The governing equation for the motion is given as: $m(d^2x/dt^2) + c(dx/dt) + kx = P_o\sin(\vartheta t)$. The general solution is obtained by adding a particular solution x_p to the solution of the homogeneous equation. The form of the particular solution is: $x_p = x_{max}\sin(\vartheta t - \varphi)$, where φ is a phase difference defined as:
$\varphi = \tan^{-1}[2(c/c_{cr})(\vartheta/\sigma)/[1-(\vartheta/\sigma)^2]$.
Magnification Factor: $f_M = x_{max}/(P_o/k) = 1/\{[1 - (\vartheta/\sigma)^2]^2 + [2(c/c_{cr})(\vartheta/\sigma)]^2\}^{1/2}$.

MOMENTS OF INTERTIA OF 3-D BODIES

Moment of Inertia of a body, with respect to a given axis is given as: $I = \int\int\int \rho\, r^2dv = \int\int\int r^2dm$ where r is the distance from the axis, ρ is the density, V is the volume and m is the mass. Generally, the moment of inertia can be expressed as $I = mR_i^2$ where R_i is the radius of gyration.

Moment of Inertia of Thin Plates.
The moments of inertia in a cartesian coordinate system x-y-z of plate thickness, t, laying in the x-y plane, are given as:
$I_x = \int\int \rho\, y^2\, dA$ $\quad I_y = \rho \int\int z^2\, dA$
$I_z = \rho \int\int r^2\, dA$ $\quad r^2 = x^2 + y^2$

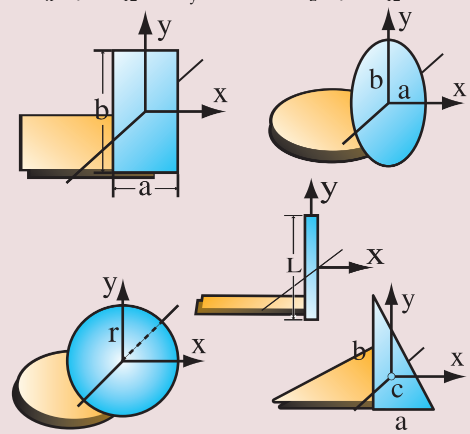
Disk:
$I_x = \rho\, r^2/_4$ $\quad I_y = \rho\, r^2/_4$ $\quad I_z = \rho\, r^2/_2$

Thin Rectangular Plate:
$I_x = \rho\, r^2/_{12}$ $\quad I_y = \rho\, r^2/_{12}$ $\quad I_z = \rho\, (a^2+b^2)/_{12}$

Thin Elliptic Plate:
$I_x = \rho\, b^2/_4$ $\quad I_y = \rho\, a^2/_4$ $\quad I_z = \rho\, (a^2 + b^2)/_4$

Thin Triangular Plate:
$I_x = \rho\, b^2/_{18}$ $\quad I_y = \rho\, a^2/_{18}$ $\quad I_z = \rho\, (a^2 + b^2)/_{18}$

Thin Rod:
$I_x = \rho\, L^2/_{12}$ $\quad I_y = 0$ $\quad I_z = \rho\, L^2/_{12}$



Moment of Inertia of Three-Dimensional Bodies

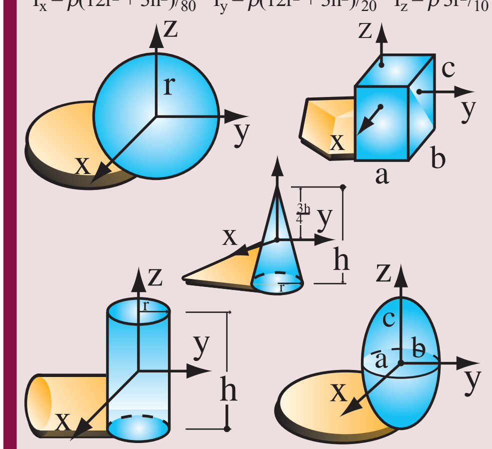
Sphere:
$I_x = \rho(2r^2)/_5$ $\quad I_y = \rho(2r^2)/_5$ $\quad I_z = \rho(2r^2)/_5$

Orthogonal Parallelepiped:
$I_x = \rho(b^2 + c^2)/_{12}$ $\quad I_y = \rho(a^2 + c^2)/_{12}$ $\quad I_z = \rho(a^2+ b^2)/_{12}$

Ellipsoid:
$I_x = \rho(b^2 + c^2)/_5$ $\quad I_y = \rho(a^2 + c^2)/_5$ $\quad I_z = \rho(a^2 + b^2)/_5$

Cylinder:
$I_x = \rho(3r^2 + h^2)/_{12}$ $\quad I_y = \rho(3r^2 + h^2)/_{12}$ $\quad I_z = \rho r^2/_2$

Cone:
$I_x = \rho(12r^2 + 3h^2)/_{80}$ $\quad I_y = \rho(12r^2 + 3h^2)/_{20}$ $\quad I_z = \rho\, 3r^2/_{10}$



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