

REVIEW

We've used the DCF method to find present value. We also know shortcut methods to solve these problems such as perpetuity present value = C/r . These tools allow us to value any cash flow including stocks and bonds. Now we will examine how to value stocks and bonds.

Bond Valuation

The central questions for a financial manager:

1. What to invest in (valuation, Time Value of Money)?
2. How to finance the investment.

Bond characteristics

- A **Bond** is a **contract** between a borrower (issuer) and a lender (bondholder). The bondholder lends money to the issuer. The issuer, in turn, agrees to pay interest on this **loan** according to a set schedule, and to repay the loan principal at a stated maturity date.

One party wants money, the other wants to lend money. Not easy for the two parties to find each other so there has been a market created to facilitate trading. The borrower issues a bond to the market, the lender has the opportunity to purchase the bond.

Bond Terms: what is the interest rate and the payment schedule.

- **Face value** (principal) of the bond is the amount to be repaid at maturity. Usually the face value is equal to \$1,000. Face value is also known as PAR Value and Principle. If the bond is sold before maturity the new owner receives the face value payment at the end of the bond term. The face value is fixed, constant over the life of the bond, this is part of the bond agreement (covenant).

- **Market value** is the price at which you could buy or sell a bond. The market value of a bond can change from day to day. The market value is determined by market forces.

MARKET VALUE IS NOT THE SAME AS FACE VALUE!

Do not confuse Face Value with Market Value !!!

- **Coupon rate** is the interest rate to be paid on the face value of a bond. The coupon rate specifies the coupon payment.

- **Coupon payment** is the periodic interest payment paid by the issuer to the bondholder. It is typically paid every six months.

$$\text{Coupon Payment} = \text{Face Value} \times \text{Coupon Rate}$$

- **Zero-coupon** bond (pure discount bond) is a bond which does not pay any interest. No coupon payments. In this case we use the discount rate to value the bond.

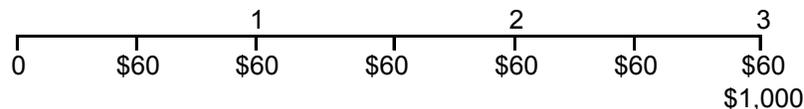
- **Perpetuity bond** (consol) is a bond that lasts forever and pays only interest. Same payment amount forever. Only need to know coupon rate, face value, and frequency. ($PV = C/r$, in this case we would be interested in C).

Bond conditions (covenant) include interest rate, term, and payment schedule. From this we know when the bond loan will be repaid.

Bonds are typically issued at face value of \$1,000.

Example

FV = \$1,000 T = 3 years Coupon Rate = 6% semi-annual payments



We need face value, coupon rate, frequency, and maturity to estimate the bonds future cash flows and estimate its value. The zero-coupon bond would require less values to solve.

Must be very careful about how the interest rate is stated. Here they say 6% semi-annual, so it's not $6\%/2=3\%$ every 6 months, it's 6% every 6

Bond Characteristics



- **Yield to Maturity**, also called the **Internal Rate of Return**, is the **average rate of return** an investor will receive from a bond if purchased at the current market price and held to maturity (YTM always expressed in annual terms).

Example:

\$1,000 Zero-Coupon Bond pays \$1,200 after 1 year. What is the YTM?

$$1000 = \frac{1200}{1+r} \Rightarrow r = 20\%$$

We do not know r but we can solve for r because we know the market value is \$1,000. So **YTM = r = 20%**.

The credit rating of a company (such as Moody's), say AAA, AA, A, BBB, BB, ..., speaks to the companies strength, the likelihood that the firm will continue to meet it's obligations. But it does not speak to the longevity of the company, it does not indicate that the company is likely to be around 30 years to repay it's long term bonds.

Here we are comparing how much we pay to how much we ultimately make. This is a key concept of bonds, used to compare values. The coupon rate stays the same over the entire life of the bond but the yield to maturity changes constantly according to the market value.

YTM IS THE DISCOUNT RATE WHICH MAKES THE BOND'S MARKET VALUE EQUAL TO THE PRESENT VALUE OF ALL FUTURE PAYMENTS OF THE BOND.

Say we are considering buying a bond. The first thing we need to do is estimate the future payments of this asset. Next we need to calculate the present value of all the future payments, for this we need the discount rate. Now we are asking ourselves "what is the proper discount rate for these future payments"? But in fact some other people have already calculated the present value of the future payments, these people are the investors in the market and the PV they calculate for all future payments is the market value of the bond because this is the fair price of the asset. So PV = Market Value of future payments. Now we ask ourselves "what discount rate did they use"? Can we find this discount rate? Yes we can by back solving the equation. This discount rate is the **YIELD TO MATURITY**. Called the **implied discount rate** because it is implied by comparing the market value to the future payments.

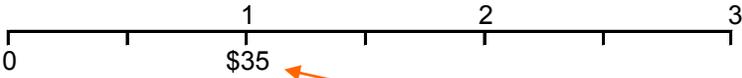
 **YTM is not the coupon rate!**

YTM is the interest rate that is required by investors in the bond market (**the market interest rate**). The market value of the bond is always known, then solve for the discount rate, this is the YTM.

YTM is the **IMPLIED DISCOUNT RATE** of the bond also known as the **“Market Interest Rate”**.

- **Current yield** is the ratio of the interest payment to the current market value of the bond (similar to YTM). Ratio of **Coupon Payment** to current **Market Value**.

Current Yield measures the IMMEDIATE RETURN we will make from the bond, not the average return.

$$\text{CURRENT YIELD} = \frac{\text{NEXT PAYMENT}}{\text{MARKET VALUE}} = \frac{35}{700}$$


Next payment with current market value = \$700

Bond Issuers

- Bonds are issued by either corporations (corporate bonds) or the government (government bonds).

- The main difference between corporate bonds and government bonds is that the **corporate issuer may default**. **Risk is the difference**, government bonds are guaranteed. The cash flow of a firm involves risk! The firm may **default**, go out of business, file for bankruptcy. But even in bankruptcy bond holders are repaid first (although maybe not all of their money).

Risk is the difference here. Government bonds are guaranteed but the cash flow of a firm involves risk. A firm may default but failing to make its bond payments usually means the firm is in bankruptcy. If the firm does go into bankruptcy the bond holders are the first to be repaid.

- **Bonds are the senior securities of a firm**. The law requires bankrupt firms to pay off their bondholders before their equity holders.

- **Government bonds** are the most reliable fixed income securities, and are considered to be risk-free.

Bond Pricing

- Value of financial securities = PV of expected future cash flows

- **Zero coupon bond**, no coupon payments, only the principle payment with interest at maturity.

$$\text{Bond Price} = \text{Present Value} = B = \frac{F}{(1+r)^T}$$

- **Example:** What is the price of a corporate zero-coupon bond that matures in 15 years if the market interest rate is 7%? The face value is \$1,000

$$B = \frac{1000}{(1.07)^{15}} = \$362.45$$

- **Perpetuity bond**, interest payments only but they continue forever.

$$B = \frac{C}{r} = \text{PV}$$

- **Example:** What is the price of a perpetuity bond that pays 7% coupon annually if the market interest rate is 8%?

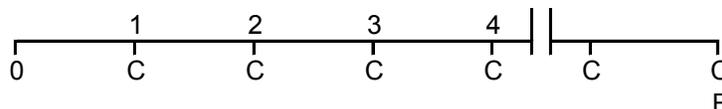
$$B = \frac{1000 \cdot .07}{.08} = \$875$$

- **Coupon bond**, interest payments at a defined interval (frequency).

F = Face Value (PAR value), C = coupon payment

$$B = \sum_{t=1}^T \frac{C}{(1+r)^t} + \frac{F}{(1+r)^T}$$

$$= \frac{C}{r} \left(1 - \frac{1}{(1+r)^T} \right) + \frac{F}{(1+r)^T}$$



- **Example:** What is the price at the time of issuance of an 8-year bond with a 9% annual coupon rate? The market interest rate for an eight-year horizon is 7%

$$C = 9\% \times 1000 = 90 \quad r = 7\% \quad B = \frac{90}{.07} \left(1 - \frac{1}{(1+.07)^8} \right) + \frac{1000}{(1+.07)^8} = \$1,119.43$$

Bond pricing

- In the examples above, the focus was on pricing the bonds given the market's required return (the market interest rate) .

- Conversely, if the bond's price is known, we can determine the market's required return, or YTM. We can find the discount rate if we have the future payments and the bond price. Why interested in this type of calculation? At the bond market we have future payments, bond price. But we do not have the discount rate, it is not part of the bond agreement. So we want to know how to calculate the discount rate. This gives us the return we will make if we buy the bond (this is also the YTM). So let's look at how to calculate the YTM.

- The **YTM** is the **discount rate** that equates the bond's **market price** to the **PV of its payments**. We solve for the discount rate so we can know the YTM at a given market price.

$$B = \sum_{t=1}^T \frac{C}{(1+r)^t} + \frac{F}{(1+r)^T}$$

- **YTM** is calculated by trial and error (calculating by hand can be very tedious use Excel or financial calculator)

Example

- Consider a bond with an 8% coupon rate that matures in 3 years. The market price is \$1,053. What is the yield to maturity of the bond?

0	1	2	3
-1053	C	C	C
	80	80	80 + F

$$1053 = \frac{80}{(1+r)} + \frac{80}{(1+r)^2} + \frac{80}{(1+r)^3} + \frac{1000}{(1+r)^3}$$

Time	Cash Flows
0	-1053
1	80
2	80
3	1080
=IRR(G9:G12, 0.0001)	
IRR(values, [guess])	

For this schedule YTM = r = 6.02%

Bond market reporting

TABLE 5.1	Bonds	Cur. Yld.	Vol.	Close		Net Chg.
Bond Market Reporting	AMF 10% 06	25.3	10	43		...
	AMR 9s16	8.8	25	102	-	.38
	ATT 5% 01	5.2	30	98.50	+	.13
	ATT 7% 02	7.1	55	100.13	-	.13
	ATT 6% 02	6.6	50	99	+	.88
	ATT 6% 04	6.9	52	97.75	+	.38
	ATT 5% 04	6.0	138	94.38		...
	ATT 7% 06	7.4	60	100.75	-	.50
	ATT 7% 07	7.6	83	101.50	+	.50
	ATT 6s09	6.7	40	89	+	.63
ATT 8% 22	8.1	97	100	+	.38	

The stock in red closed at 100.75% of it's Face Value, closed at a PREMIUM.
 The stock in blue closed at 89% of it's face value, at a DISCOUNT.
 The bottom stock closed at 100% of it's face value, PAR.

In the quote **ATT 8½ 22** the 8½ is the **annual** coupon rate (the rate is stated as annual, if the coupon is paid every 6 months the 6 month coupon rate will be $8.5/2=4.25\%$). The 22 is the year in which the bond matures.

Current Yield is the **YTM** of the bond.

Volume tells how many of that issue changed hands that day.

Close is the last trading price of that day in percentage of face value. We know that **MARKET PRICE = CLOSE PRICE (%) * FACE VALUE**.

The + or - indicates if the direction of movement since that previous trading day.

Net Change is the change in the bonds price in percent since that previous trading day.

Note the AMF stock at the top, it's current yield is 25.3%. That is saying that the bond is VERY HIGH RISK. The high return is intended to compensate for that risk.

In this table we see that for ATT the current yield (it's not YTM) is kind of increasing with maturity. Although we have different coupon rates we can still identify this pattern because for bonds that are paid in the future there is more risk that the firm will go bankrupt even though coming down from 99% to 97.75% (year 02 to 04) is not too much there is still some increase in the discount rate even for a firm like AT&T.

The current yield also varies because the maturity is different.

The relation between bond prices and interest rates

- Fundamental fact:

Bond prices move inversely to interest rates!

One reason for this is opportunity cost. If your money is tied up in a bond you will lose the opportunity to make other investments. The discount rate relates to a similar investment available in the market. The investors must be compensated for this. If the current market interest rates are very high then the opportunity cost will be very high and the discount rate for investing in this bond will be high and this means that the market value of the bond will be LOW. So with increasing interest rates the bond prices are going down (to attract buyers).

This does not mean that the magnitude of the change in every bonds price will be the same, in fact they will not be the same. For a given change in interest rates the change in bond price will vary greatly, some will change a little and some will change a lot. **So what effects the magnitude of the change in the bond price for a given change in interest rate?** What property of the bond effects the sensitivity of it's price to changes in interest rates.

We examine an example. We have two zero-coupon bonds and the market interest rate changes from 9% to 12% (table below).

Example:

Determine the market value of a corporate bond with 20 years to maturity that has a 9% coupon rate, if the market interest rate is 7%, 9%, 11%.

Interest
Rate

7%: $B = \frac{90}{.07} \left(1 - \frac{1}{1.07^{20}}\right) + \frac{1000}{1.07^{20}} = \$1,211.88$

9%: $B = \frac{90}{.09} \left(1 - \frac{1}{1.09^{20}}\right) + \frac{1000}{1.09^{20}} = \$1,000$

11%: $B = \frac{90}{.11} \left(1 - \frac{1}{1.11^{20}}\right) + \frac{1000}{1.11^{20}} = \840.73

Bond
Value

PREMIUM

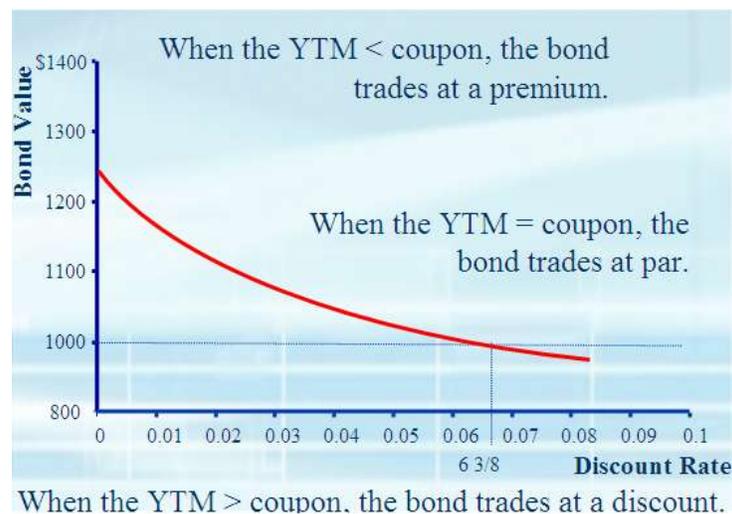
DISCOUNT

We see that the present value of the bond goes down as the interest rate increases!

Also, when the coupon rate = market rate (discount rate) the face value equals the present value. This indicates that the money is growing at the same rate that we discounted it back to the present at.

Although the discount rate and the coupon rate are different we can learn something by comparing them:

- When coupon rate = Interest rate (YTM),
the bond's price = face value (**PAR bond**)
- When coupon rate > Interest rate (YTM),
the bond's price > face value (**Premium bond**)
- When coupon rate < Interest rate (YTM),
the bond's price < face value (**Discount bond**)



Back to our question:

- What property of a bond affects the sensitivity of its price to the interest rate?
For **zero-coupon** bonds the answer is **MATURITY**. (see example below)

Example 1:

The price of which of the following two **zero-coupon** bonds will be affected more if the interest rate increases from 9% to 12%?

	Bond A	Bond B
Face Value	\$1,000	\$1,000
Time to Maturity	2 years	10 years
Price (YTM = 9%)	\$841.68	\$422.41
Price (YTM = 12%)	\$797.19	\$321.97

As the interest rate changes the longer maturity bonds will feel the effect more, the rate change will have a longer time to act on the value. So in this example the long term bond losses more value as the interest rate increases.

But what about the general case? Consider the example below:

The relation between bond prices and interest rates

Example 2:

The price of which of the following bonds will be affected more if the interest rate increases from 9% to 12%?

If we only consider the years to maturity we think the 12 year bond will be more sensitive. But one of these bonds has a **coupon payment!** How will this effect the bonds value?

	Bond A	Bond B
Face Value	\$1,000	\$1,000
Coupon Rate	15%	0%
Time to Maturity	12 years	10 years
Price (YTM = 9%)	\$1429.64 (143%)	\$422.41 (42.2%)
Price (YTM = 12%)	\$1185.83 (118.6%)	\$321.97 (32.2%)

The form of the solution is: 9%: $B_A = \frac{150}{.09} \left(1 - \frac{1}{(1+.09)^{12}}\right) + \frac{1000}{(1+.09)^{12}} = \$1,429.64$

If we have zero-coupon there is only one payment at the end, this is the entire value of the bond. But if we have many payments before maturity we get the value out closer to time zero, this helps offset the effect of a coupon rate which is lower than the market rate. In percentage terms bond B has suffered more. But **maturity is not enough to establish a general rule**. We see here that bond A had a longer maturity but the change was lower. What we see here suggest that we may want to look at the **AVERAGE MATURITY...**

Duration

- The duration of a bond is the **PV-weighted average of the maturities of the individual bond's payments:**

$$D = \frac{1}{B} [1 * PV(C_1) + 2 * PV(C_2) + \dots + T * PV(C_T)]$$

$$D = \frac{1}{B} \left(\sum_{t=1}^T \frac{t * C}{(1+r)^t} + \frac{T * F}{(1+r)^T} \right)$$

This CT includes all the payments at time T, **coupon and final.**

The duration measures the degree to which the market price of the bond is sensitive to changes in the market interest rate.

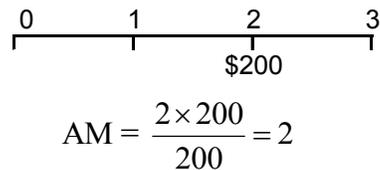
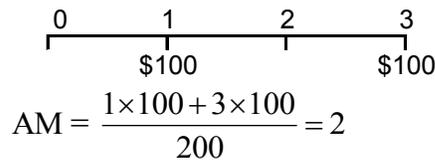
To ultimately answer the question about sensitivity we need weighting factors valued according to the present value. This is called the Duration of the Bond.

AM = Average Maturity

AM = Average Maturity

Example:

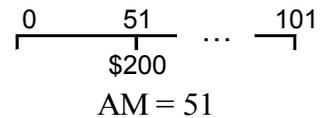
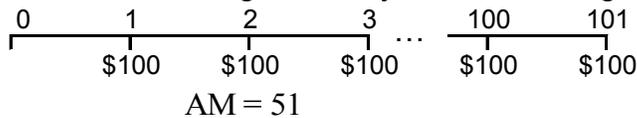
Consider the average maturity of the following two bonds:



On the left the dominant payment is year 1 because it is closest to time 0. Very different bonds but they have the same sensitivity.

Example:

Consider the average maturity of the following two bonds:



The present value of A is more dependent on payment 1, not some payment 100 years from now! (The present value of some payment 100 years from now is going to very low, close to 0). Getting the money closer to time zero helps negate the effects of interest rate changes. The long term bond is always going to suffer more. To improve our estimating method we must include weights wrt time and PV. Time is what is having an effect on PV so we compensate for it.

Example

- What is the duration of a (\$1,000) bond with 3 years to maturity that has a 9% coupon rate, if the interest rate is 8%?

First find B:

$$B = \frac{90}{.08} \left(1 - \frac{1}{(1+.08)^3}\right) + \frac{1000}{(1+.08)^3} = \$1,025.77$$

Now find D:

$$D = \frac{1}{1025.77} \left[\left(1 \times \frac{90}{(1+.08)^1}\right) + \left(2 \times \frac{90}{(1+.08)^2}\right) + \left(3 \times \frac{90}{(1+.08)^3}\right) + \left(3 \times \frac{1000}{(1+.08)^3}\right) \right] = 2.76$$

This is a 3 year bond but it's duration is only 2.76 years!

Dominant Payment

Intuitively: we have a coupon rate which is HIGHER than our discount rate so we expect to make our money sooner, the duration will be less than the maturity.

Duration

$$D = \frac{1}{B} [1 * PV(C_1) + 2 * PV(C_2) + \dots + T * PV(C_T)]$$

$$D = \frac{1}{B} \left(\sum_{t=1}^T \frac{t * C}{(1+r)^t} + \frac{T * F}{(1+r)^T} \right)$$

- **A zero-coupon bond has a duration equal to its maturity**

(we can see this in the duration equation, get T*B/B)

- **Coupon bonds have durations shorter than their maturity**

(see example above, we get some PV before maturity so weighted average is lower)

- **Duration increases with maturity**

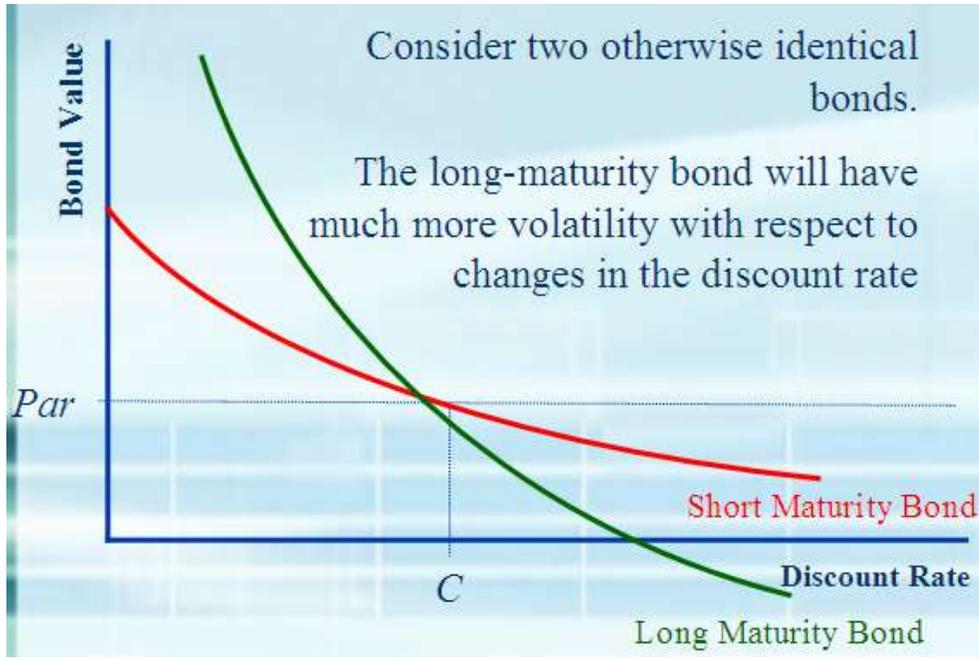
(holding everything else constant)

- **Duration is inversely related to the coupon**

Zero-Coupon has duration = Maturity. A Coupon bond has payments earlier (closer to present) so the average maturity will go earlier, closer to the present.

- **Duration is inversely related to the Yield To Maturity**

(In the formula we can see the YTM is in the denominator, so as it increases the duration decreases.) The higher the YTM the lower the bond price. The total effect will be lower duration.



Predicting bond price changes using duration

Why do we need a measure like Duration? We can compare the effects of a change in interest rates between different bonds. Two bonds, A and B, how will each bond be effected by a change in the interest rates? Say interest rates go up, bond A has longer duration, which bonds price will increase more?

Bond A: Duration = 4.2, Bond B: Duration = 1.7

Market interest rate goes up. The price of which bond will drop more? Bond A's price will drop more. It is going to be tying up our money in the market longer paying a discount rate lower than the market rate. It (bond A) must compensate the investor by lowering it's price.

THE LONGER THE DURATION THE MORE SENSITIVE THE BOND IS TO CHANGES IN THE INTEREST RATE.

- The approximate change in the price of a bond given the change in yield can be determined as follows:

$$\Delta B = -D \frac{B}{(1+r)} \Delta r$$

This denominator r is the original r before the price change.
The sign of Δr indicates increase (+) or decrease (-) in interest rate.

Using duration to measure the response of the bonds price to a change in interest rates. Notice the effect on bond price due to interest rate change is the same for a zero-coupon bond as it is for a coupon bond. The formula above is dealing with duration which is the weighted average of the maturities.

Example:

Consider an 8% bond with 25 years to maturity. Assume that the interest rate is 6%. The duration of this bond is 13.55. How much will the bond's price change if the market interest rate decreases by two basis points to 5.98%?

$$D = 13.55 \quad \Delta r = .0598 - .06 = -.0002$$

$$B = \frac{80}{.06} \left(1 - \frac{1}{1.06^{25}}\right) + \frac{1000}{1.06^{25}} = 1256 \quad \Delta B = (-13.55) \frac{1256}{1.06} (-.0002) = 1256$$

Positive change for decreasing interest rate due to inverse relationship between bond prices and interest rates. D represents the weighted average of the maturities. In the above example:

$$D = \frac{1}{1256} \left(\frac{1 \times 80}{1.06} + \frac{2 \times 80}{1.06^2} + \dots + \frac{25 \times 80}{1.06^{25}} + \frac{25 \times 1000}{1.06^{25}} \right) = \$13.55$$

Duration creates a measure which is valid for coupon and non-coupon bonds.

APR and bond equivalent yield

- The YTM on T-bonds and T-notes is calculated as a semi-annual rate because the coupons are paid semiannually

YTM calculated in semi-annual terms but then doubled to get annual.

- The financial press multiplies this number by two, and reports the **Annualized Percentage Rate (APR)** (easier to compare in annual terms).

So APR is a nominal rate because it does not take into account compounding.

- The **APR** is also called the **Bond Equivalent Yield**.

Two bonds:

Bond A: 5% semi-annual (has 10% APR)

Bond B: 10% annual

Bond A has the greater return due to compounding.

Nominal and Effective Interest Rates (EAR)

- **Effective Annual Interest Rate (EAR)** is the actual rate generated by the investment. It results from adjusting the **nominal annual interest rate** to compounding during the year

$$r_{\text{effective}} = \left(1 + \frac{r_{\text{nominal}}}{m} \right)^m - 1$$

where **m** is the number of compound intervals per year

- Given a constant nominal rate, the effective rate increases with **m** (number of compounding periods per year)

- If **m** becomes very large (infinite), then

$$r_{\text{effective}} = e^{r_{\text{nominal}}} - 1 \quad \text{where } e \approx 2.71$$

Continuous compounding

**MUST
KNOW e
VALUE FOR
EXAM**

Effective Annual Yield

- The **Effective Annual Yield** adjusts the APR for the effects of compounding.
(We adjust the APR to the number of payments every year.)
- With semi-annual compounding:

$$EAY = \left(1 + \frac{APR}{2}\right)^2 - 1$$

general form: $EAY = \left(1 + \frac{APR}{\# \text{ payments per yr}}\right)^{\# \text{ payments per yr}} - 1$

[These types of rate adjustments are used to calculate investment income but not used in PV calculations? Is this because of the units of the equation? Time base?]

Example:

If a bond has a semi-annual YTM of 3%, then After 1 year, each dollar invested grows to (APR = 2 * 3% = 6%)

$$\$1 \times (1.03)^2 = \$1.0609$$

In this case APR = 6% and $EAY = \left(1 + \frac{6\%}{2}\right)^2 - 1 = 6.09\%$ per year. Effectively this bond returns 6.09% every year.

- Thus, the **APR is 6%**, but the **Effective Annual Yield is 6.09%**

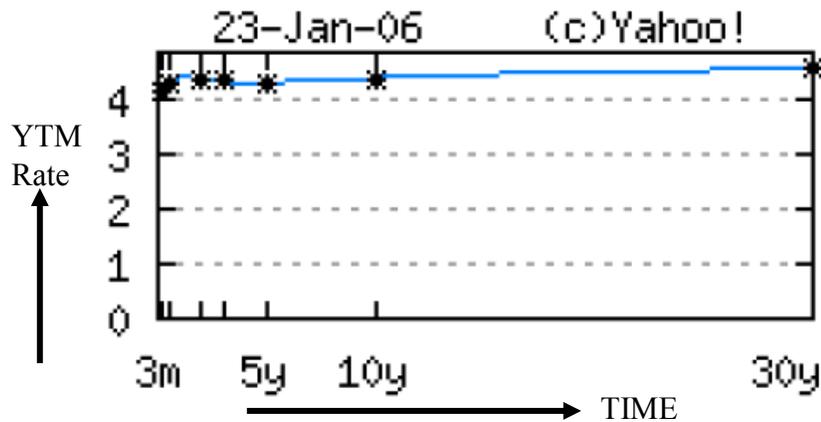
The term structure of interest rates

- So far we assumed that the interest rate is the same for all maturities (the yield curve is flat) (we've assumed the interest rate is constant for every period).
- However, in reality, at any point in time, bonds with different times to maturity will have different yields to maturity
- The pattern of interest rates for bonds with different maturities at a given point in time is called the **Term Structure of Interest Rates**.

There is no single risk free rate in the market. Each term/period/maturity has it's own rate. The interest rate of a bond is a function of maturity called the **Term Structure of Interest Rates**. If someone ask "What is the current risk free rate in the market"? the answer will not be a single number, would be a rate and it's

time period. Interest rate is a function of maturity called the **TSIR**.

Term Structure of Interest Rates Curve (Yield Curve):



Typically we will see the rate increase with time to compensate for opportunity loss. In this example we see the short term rates dipped between 3 months and 5 years. One explanation could be that demand for long term (30 yr) bonds fell off while demand for short term bonds increased. The increased demand would have the effect of lowering the rate of return.

The demand for longer maturity bonds (such as 30 years) is much lower than that for short term bonds. So the return on these longer term bonds must be higher.

Practice questions

- 5.2
- 5.5
- 5.7
- 5.9
- 5.10