Risk and Return and Portfolio Theory

Intro:
Last week we learned how to calculate cash flows, now we want to learn how to discount these cash flows. This will take the next several weeks. We know discount rate represents risk. A firm with high risk will offer higher return. Say we are interested in investing $1000 in either Microsoft or GE. We may consider Microsoft more risky so we want a higher return to compensate. So can we just derive the risk of each firm in the market and value the stocks accordingly? It's not that simple. Investors usually hold a portfolio of several different stocks and other assets. Now the relevant question is what is the contribution of a particular stock to the risk level of the portfolio? We will find that the risk added to the portfolio by a particular stock does not always coincide to the standard deviation of the stock which was added. Other factors will effect the portfolio risk.

RELEVANT RISK IS MEASURED BY THE EFFECT OF ADDING A PARTICULAR STOCK TO THE RISK OF THE PORTFOLIO.

To measure the relevant risk we need to know something about portfolio theory. How investors choose the assets in the portfolio. And then asses the contribution of adding each individual stock to the general risk of the portfolio. Once we know these things we will bring in an analysis of the cost of capital. The **General Asset Pricing Model** takes the contribution or the effect of adding each stock to the risk of the portfolio to the cost of capital of the individual (?).

Basic Statistics

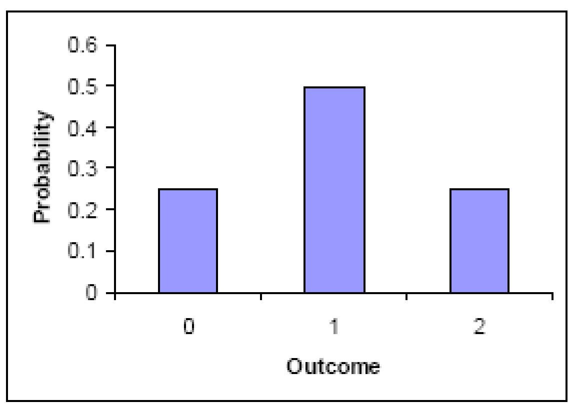
Random variable: a variable that takes different values with some probabilities

Example: a random variable X can take the following realizations:

- 0 with probability 0.25
- 1 with probability 0.5
- 2 with probability 0.25

Sum of all possible probabilities of a particular random variable is 1.

Probability Distribution: a schedule of all possible realizations and respective probabilities



Basic Statistics

Expected value (mean): expected realization of a random variable:

$$E(X) = \sum_{i=1}^{n} X_i P_i$$

where **X_i is the outcome in state i** and **P_i is the probability of state i**

We have a schedule of outcomes and their probabilities. We need a method to measure their likelihood. Expected value is the average outcome, it gives us an idea of the size or magnitude of the random variable.

-Consider the example from the previous page

E(x) = 0*.25 + 1*.5 + 2*.25 = 1

Variance: expected value of squared distance from the mean:

$$\sigma^2(X) = \sum_{i=1}^{n} (X_i - E(X))^2 * P_i$$

Variance gives us an idea of the spread of the distribution. We are measuring the distance from the mean. We square the distance so we know it's positive. The reason for using square and not absolute value is that absolute value has some mathematical shortcomings. For instance it can be impossible to differentiate in some circumstances.

Standard Deviation: square root of variance:

$$\sigma(X) = \sqrt{\sum_{i=1}^{n} (X_i - E(X))^2 * P_i}$$

Taking the square root gives us back the scale of the measurement of distance from the mean.

Xi	Pi
0	0.25
1	0.5
2	0.25

$$\sigma^2 = (0-1)^2 * .25 + (1-1)^2 * .5 + (2-1)^2 * .25 = .5$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{.5} = .707$$

These are the two major measures when dealing with one random variable.

Basic Statistics

Covariance: measure of co-movement between two random variables

$$\text{Cov}(X,Y) = \sum_{i=1}^{n} (X_i - E(X)) * (Y_i - E(Y)) * P_i$$

Positive covariance means the variables move in the same direction, positively related. Covariance is a measure of the direction of correlation, positive or negative. The magnitude of covariance is not too important.

Correlation coefficient: standardized measure of the co-movement between two random variables:

$$\rho_{X,Y} = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{\sum_{i=1}^{n} (X_i - E(X)) * (Y_i - E(Y)) * P_i}{\sqrt{\sum_{i=1}^{n} (X_i - E(X))^2 P_i * \sum_{i=1}^{n} (Y_i - E(Y))^2 P_i}}$$

Measures the strength of the relationship between two random variables (in a normalized fashion). The range of correlation is $-1 \le \rho \le 1$. Since correlation coefficient is a function of covariance it includes both the direction of movement between the variables and the strength of the relation.

-**Covariance** and **correlation** tell us whether two random variables tend to **move** in the **same** or the **opposite** direction.

-The **correlation** between two random variables will always be between -1 and 1. If the correlation is 0, we say that the variables are uncorrelated. If it is 1, then the variables are perfectly correlated, and if it is -1, then the variables are perfectly negatively correlated

Summer Beach Population

Prob	Temp	# People
1/3	60	800
1/3	70	1000
1/3	80	1200

Cov(temp, peop) = (60-70)(800-1000)(1/3) + (70-70)(1000-1000)(1/3) + (80-70)(1200-1000)(1/3) = (neg)(neg) + (0)(0) + (+)(+) = Positive number

Swap the 800 and 1200 and we will see a negative covariance.

Example

-Calculate the covariance and the correlation coefficient between X and Y

State	1	2	3
X	2	1	0
Y	0	1	6
Probability	0.25	0.5	0.25

Result Number

By inspection we expect negative correlation (covariance).

$$E(x) = \sum_{i=1}^{n} X_i P_i$$

$E(x) = 2*.25 + 1*.5 + 0*.25 = 1$

$E(y) = 0*.25 + 1*.5 + 6*.25 = 2$

$$\sigma^2(x) = \sum_{i=1}^{n} (X_i - E(x))^2 * P_i$$

$$\sigma^2(x) = (2 - 1)^2*.25 + (1 - 1)^2*.5 + (0 - 1)^2*.25 = .5$$

$$\sigma^2(y) = (0 - 2)^2*.25 + (1 - 2)^2*.5 + (6 - 2)^2*.25 = 5.5$$

$$\sigma(x) = \sqrt{\sum_{i=1}^{n} (X_i - E(x))^2 * P_i}$$

$\sigma(x) = \sqrt{\sum_{i=1}^{n} (X_i - E(x))^2 * P_i} = \sqrt{.5} = .707$

$\sigma(y) = \sqrt{\sum_{i=1}^{n} (Y_i - E(y))^2 * P_i} = \sqrt{5.5} = 2.345$

$$\text{Cov(X, Y)} = \sum_{i=1}^{n} (X_i - E(x)) * (Y_i - E(y)) * P_i$$

$$\text{Cov(X, Y)} = (2-1)*(0-2)*.25 + (1-1)*(1-2)*.5 + (0-1)*(6-2)*.25 = -1.5$$

$$\rho_{X,Y} = \frac{\text{Cov(X, Y)}}{\sigma_x \sigma_y} = \frac{-1.5}{.707*2.345} = -0.90475$$

Calculating rates of return

-Stock returns are made up of two components:

Beginning Value = Asset Price

Ending Value = Dividend + Current Price

P_{t-1} = Purchase Price P_t = Current Price

- Dividends
- Changes in the price of stock (capital gain)

$$r_t = \frac{\text{Ending value} - \text{Begining value}}{\text{Begining value}}$$

$$= \frac{(D_t + P_t) - P_{t-1}}{P_{t-1}} = \frac{D_t}{P_{t-1}} + \frac{P_t - P_{t-1}}{P_{t-1}}$$

Separate dividend from capital gain.

$$= \text{Dividend yield} + \text{Capital gain}$$

Example
Suppose an investor purchases a share of IBM common stock for \$100 on January 1. The stock pays \$4 in dividends on December 31, and its market price on December 31 is \$108

-The **Rate of Return** on this investment is

$$r_t = \frac{D_t}{P_{t-1}} + \frac{P_t - P_{t-1}}{P_{t-1}} = \frac{4}{100} + \frac{108 - 100}{100} = \frac{12}{100} = 12\%$$

So we can calculate the return on stocks which we have purchased in the past. But we have no return calculation which looks forward! The forward return is a random variable. Random variables are characterized by distributions. Probability distributions are summarized by the above major measures. So it can be useful to speak of expected values and standard deviations of returns and correlation if two random variables.

Calculating Rates of Return

-Rates of return for most assets are random

-Looking backward, we can calculate realized rates of return

-Looking forward, we characterize **random events** with probabilities and probability distributions

-Probability distributions are often described by summary measures, such as mean and variance

-Expected return and standard deviation are often derived from historical data

Looking forward the price of the stock is a random variable (random walk). With multiple stocks we look at correlation coefficient and other measures.

Investor Preferences

- We will make two fundamental assumptions about investor preferences

• Investors prefer more wealth to less (other things equal, investors prefer higher expected returns)

• Investors are risk averse (other things equal, investors prefer a lower standard deviation of their wealth)

Want low standard deviation of the return because standard deviation represents risk. This implies investors want to avoid risk. But some investors will take on more risk for a chance at higher return.

Risk Premium is the difference in return the investor requires in order to take on a more risky asset. In practice (actual measurement of real investors) the risk premium is about 8%.

The price of a risky asset which does not return more than a risk free asset has to fall.

Consider $r_t = \frac{D_t}{P_{t-1}} + \frac{P_t - P_{t-1}}{P_{t-1}}$. As P_{t-1} falls the value of the investment increases.

Risk Premium

In the beginning investors wanted only to put their money in one stock thinking it would be a "winning stock" meaning high return and low standard deviation.

-Because investors are risk averse, they require a higher return for bearing risk. In other words, they require a risk premium. We can decompose the return on a security into the risk-free component and the risk premium:

Total return = Risk-free return + Risk premium

-Historically, the market for common stocks has commanded a risk premium of approximately 8%. This means that investors require, on average 8% above the risk-free rate in order to induce them to invest in common stocks instead of in T-bills. **Note: the "risk premium" is the 8%, the percentage above the risk-free return at which the risk adverse investor will take the investment.**

To the investor a "good stock" is one with low risk and high return.

Modern Portfolio Theory

-Before **Harry Markowitz pioneered portfolio theory** and Eugene Fama and others pioneered the efficient markets theory, investment analysis focused on picking winners in the stock market Eugene Fama found that, holding everything else constant, it is almost impossible to find stocks which have relatively high expected return. This is because the market is efficient in processing information. If one stock shows very high expected return compared to another stock with the same risk, the demand for that stock will be very high which will increase the price of that stock which diminishes the return until the abnormally high expected return is gone and the return is similar to other comparable risk stocks. Abnormal profits immediately disappear in an efficient market.

-Because the investment focus was on picking winners, risk was usually measured by the **stock - return variance** or **stock-return standard deviation**

	X	Y
Std Dev	30%	30%
E(R)	12%	20%

-One of Harry Markowitz' important contributions was to recognize that rational investors hold diversified portfolios to minimize their risk. Markowitz found that it was not very important to look at the standard deviation of the stock because if we hold many different stocks then the risk of the many stocks can be reduced significantly. The risk can be diversified away if we buy many stocks. This leads to the idea that although stock-return standard deviation measures the risk of a security if you hold it alone, it is not very informative about how the security contributes to the riskiness of a diversified portfolio. Both stocks have the same std dev (risk). Investors will prefer stock Y which has expected return of 20%. But this higher return will increase demand which will raise the price of the asset and thereby decrease the earnings. This is the principle of the efficient market. Diversification (buying many different kinds of stocks) reduces risk.

Now we have to ask ourselves how adding a stock to the portfolio influences the risk of the portfolio. This is the RELEVANT RISK.

Portfolio Weights

The measurement of relevant risk, the contribution of a stock to the risk level of the portfolio, will ultimately determine the cost of capital of the firm. So we consider the risk of the portfolio, not of single assets. So we must know the portfolio we will hold and then asses the contribution of each stock to the risk of the portfolio.

Portfolio Theory:
- How investors choose assets
- How the properties of each asset effect the properties of the portfolio

-**A portfolio is uniquely defined by the portfolio weights**

-Suppose there are N assets, i=1, 2, …, N

-Then, the portfolio weight, W_i, is

$$W_i = \frac{\text{Amount Invested in Asset i}}{\text{Total Amount Invested}}$$

A **PORTFOLIO** is a collection of many assets defined by the weights of the assets where weight is the relative size of the investment in the particular asset (above).

Portfolio Expected Return

(Here **R** is Return)

-The portfolio expected return is equal to the weighted average of the returns on the individual assets in the portfolio, where the weights are given by the portfolio weights

-With 2 assets, the portfolio expected return is

$$E(R_P) = W_1E(R_1) + W_2E(R_2)$$

-With N assets, the portfolio expected return is

$$E(R_P) = \sum_{i=1}^{N} W_iE(R_i)$$

Portfolio Variance

-The portfolio variance depends on both the variance of the individual assets in the portfolio and their covariance

-With 2 assets, the portfolio variance is (derived from the covariance matrix below)

$$\sigma_P^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\mathrm{Cov}(R_1,R_2)$$

$$= W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\rho_{1,2}\sigma_1\sigma_2$$

Where $\rho = \frac{\mathrm{Cov}(X, Y)}{\sigma_x\sigma_y}$

What is the expected value and standard deviation of the portfolio?

Expected value of the portfolio is the weighted average of the return of each asset.

Variance of portfolio is a function of the variance of individual assets and the covariance of the assets.

-With N assets, the portfolio variance is

$$\sigma_P^2 = \sum_{i=1}^{N} W_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} W_iW_j\mathrm{Cov}(R_i,R_j)$$

$$\sigma_P^2 = \sum_{i=1}^{N} W_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{\substack{j=1 \\ j\neq i}}^{N} W_iW_j\rho_{i,j}\sigma_i\sigma_j$$

Diagonal terms all other terms

$j \neq i$ says we are skipping the diagonal terms in matrix below.

The Covariance Matrix

		W_1	W_2	W_3	…	W_N
	INDEX	1	2	3	…	N
W_1	1	σ_1^2	$\sigma_{1,2}$	$\sigma_{1,3}$		$\sigma_{1,N}$
W_2	2	$\sigma_{2,1}$	σ_2^2	$\sigma_{2,3}$		$\sigma_{2,N}$
W_3	3	$\sigma_{3,1}$	$\sigma_{3,2}$	σ_3^2		$\sigma_{3,N}$
…	…					
W_N	N	$\sigma_{N,1}$	$\sigma_{N,2}$	$\sigma_{N,3}$		σ_N^2

Diagonal is just the variances, all other cells are the covariance's. Now say the cells are weighted, we would write the sum of all cells as (for a 2x2 matrix):

$$\sigma_1^2 w_1 w_1 + \sigma_{1,2} w_1 w_2 + \sigma_{2,1} w_2 w_1 + \sigma_2^2 w_2 w_2$$

We ca rewrite this as:

$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{1,2} \quad \text{where} \quad \sigma_{x,y} = \sigma_{y,x}$$

So we are summing all cells where each cell is weighted. All of this derives the variance of the portfolio.

So we ask, which is more dominant in effecting the riskiness of the portfolio, variance or covariance? That is to say, the standard deviation of the individual stock or the standard deviation between stocks? Covariance is more strong, there are many more covariance terms than there are variance terms. As N increases and we have many assets we have many more covariance's. For N assets we have N^2 cells in the covariance matrix. N of these are variances and $N^2 - N$ are covariance terms.

There are N^2 terms of which **N are variances** and $N^2 - N =$ **N(N-1) are covariance's**.
N variances and N2 terms, take the percentage of the variances.

$\% \text{ Variances} = \frac{N}{N^2} = \frac{1}{N}$. Now as N goes to ∞ the weight of the variances goes to 0.

The importance of the variances goes to zero as N goes to infinity. So **covariance is much more important**.

$\% \text{ Covariance} = \frac{N^2 - N}{N^2} = \frac{N(N-1)}{N} = N - 1$. So covariance stays large as N goes to ∞.

The relative importance of Variance and Covariance

-Suppose the portfolio is diversified, $W_i \approx \frac{1}{N}$, and large $N \to \infty$. Then

$$\sigma_P^2 = \sum_{i=1}^{N} W_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} W_i W_j \sigma_{i,j} \approx \sum_{i=1}^{N} \left(\frac{1}{N}\right)^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \left(\frac{1}{N}\right)^2 \sigma_{i,j}$$

$$= (1/N)\left(\frac{1}{N} \sum_{i=1}^{N} \sigma_i^2\right) + (1 - 1/N)\left(\frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \sigma_{i,j}\right)$$

$$= (1/N)\overline{\text{Var}} + (1 - 1/N)\overline{\text{Cov}} \approx \overline{\text{Cov}}$$

The **covariance** of a new stock added to a portfolio is what effects the overall risk of the portfolio.

Here we are seeing that the standard deviation does not always coincide with the relevant risk, there are other stronger factors such as the covariance with other stocks in the portfolio.

To know what is the relevant risk of each stock allows us to know how to convert this risk to the cost of capital, we need to know the portfolio we are working with, what it's characteristics are. That is what we have done above, derived measures for the portfolio.

The effect on the portfolio of a new stock is the **covariance** of the new stock with all the other stocks in the portfolio.

Ultimately we will choose the portfolio members based on the covariance.

Example

-Compute the expected returns and variances for a portfolio consisting of the following securities:

Outcome	Boom	Normal	Recession
Probability	0.3	0.5	0.2
Return on stock 1	20%	10%	0%
Return on stock 2	12%	5%	-2%

$$E(x) = \sum_{i=1}^{n} X_i P_i \qquad \sigma^2(x) = \sum_{i=1}^{n} (X_i - E(x))^2 * P_i \qquad \sigma(x) = \sqrt{\sum_{i=1}^{n} (X_i - E(x))^2 * P_i}$$

$$Cov(X, Y) = \sum_{i=1}^{n} (X_i - E(x)) * (Y_i - E(y)) * P_i \qquad \rho_{X,Y} = \frac{Cov(X, Y)}{\sigma_x \sigma_y}$$

Say our portfolio has $150 in stock 1 and $100 in stock 2.

What is E(R) of the portfolio? First we must calculate the weights:

$$W_1 = \frac{150}{150+100} = \frac{150}{250} = .6 \qquad W_2 = \frac{100}{150+100} = \frac{100}{250} = .4$$

$$E(R_1) = .20*.3 + .10*.5 + 0*.2 = .11 \qquad E(R_2) = .12*.3 + .05*.5 - .02*.2 = 5.7\%$$

Then… $E(R_p) = W_1E(R_1) + W_2E(R_2) = .6*.11 + .4*.06 = 9\%$

Now find the variance and standard deviation of the portfolio:

$$\sigma^2_{R_1} = (.2 - .11)^2*.3 + (.1 - .11)^2*.5 + (0 - .11)^2*.2 = .49\% \qquad \sigma_{R_1} = \sqrt{.0049} = 7\%$$

$$\sigma^2_{R_2} = (.12 - .057)^2*.3 + (.05 - .057)^2*.5 + (-.02 - .057)^2*.2 = .24\% \qquad \sigma_{R2} = \sqrt{.0024} = 4.9\%$$

$$Cov(X, Y) = (.2-.11)*(.12-.057)*.3 + (.1-.11)*(.05-.057)*.5 + (0-.11)*(-.02-.057)*.2 = .343\%$$

$$\rho_{1,2} = \frac{.343\%}{7\% * 4.9\%} = 1 \quad \text{PERFECT CORRELATION !}$$

$$\sigma_P^2 = \sum_{i=1}^{N} W_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} W_i W_j \sigma_{i,j} \approx \text{Cov}$$

$$\sigma^2_{port} = .6^2*.0049 + .4^2*.0024 + 2*.6*.4*.0034 = .0038 = .378\% \approx Cov(R_1, R_2) = .34\%$$

$$\sigma_{port} = \sqrt{.00378} = 6.15\%$$

Goal is to derive the optimal portfolio. First we will learn how the properties of single assets effect the properties of the portfolio. Begin by looking at some extreme cases.

Perfect POSITIVE Correlation

- Suppose $\rho = 1$. Then

$$\sigma_P^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\rho_{1,2}\sigma_1\sigma_2$$

= 1 when perfect

$$= W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_1\sigma_2$$

$$= (W_1\sigma_1 + W_2\sigma_2)^2$$

(Factored)

leads to …

$$\sigma_P = W_1\sigma_1 + W_2\sigma_2$$

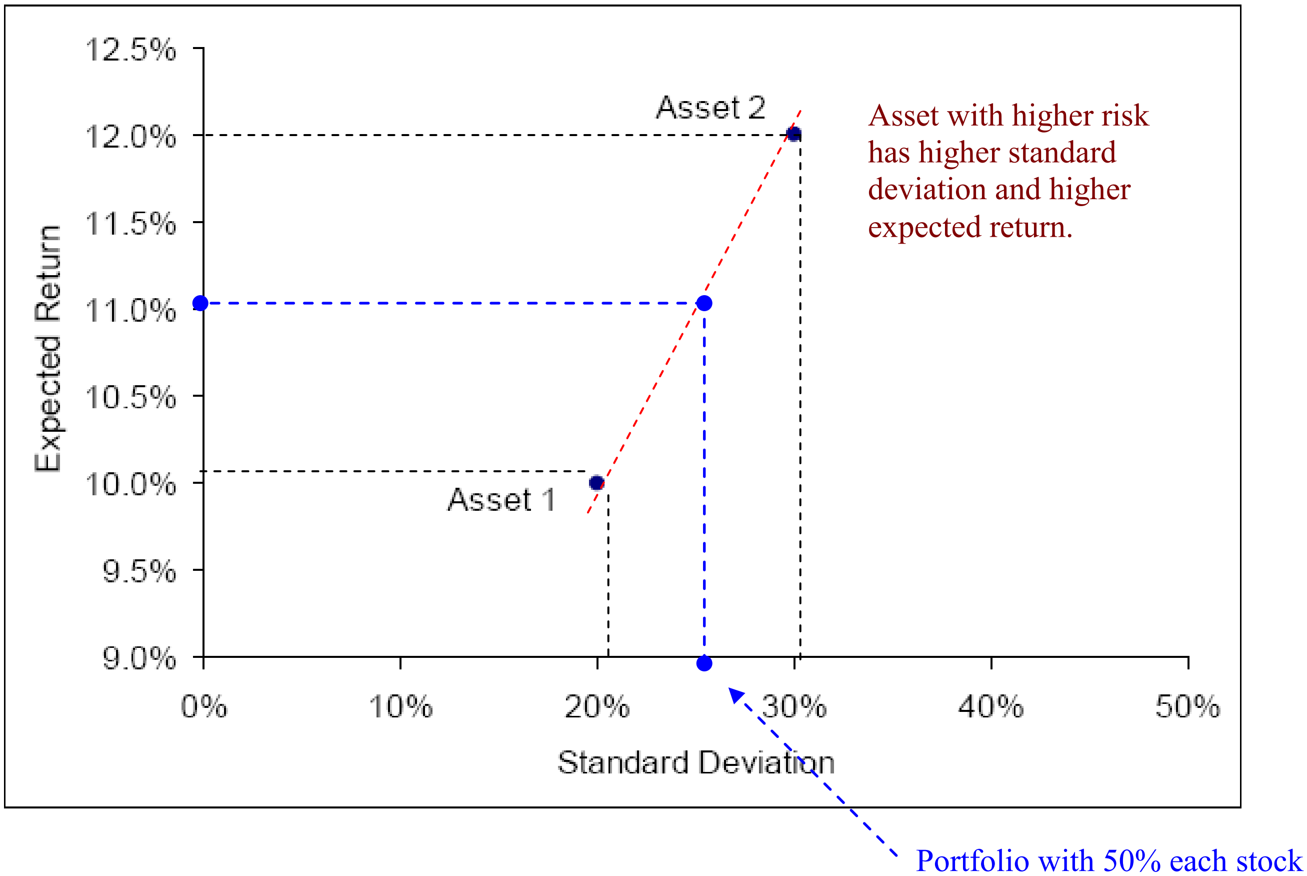
Here we have a case of **perfect positive correlation**. Is it good for us or bad for us?

The result shows that the risk of the portfolio (std dev) is the weighted average (according to amount invested) of the individual standard deviations. Even if we adjust our weights (alter the amounts invested) we still cannot reduce the risk below that of the lowest std dev in the portfolioi.

So the standard deviation of the portfolio (with perfect correlation) is at least as high as the risk of the lower risk asset. Perfect correlation does not help us much.

Perfect POSITIVE Correlation

Here we find that we cannot reduce the risk below that of asset 1.



Expected return is independent of correlation. Our 50% will have the mid-point expected return. This says the portfolio we can construct from these two perfectly correlated stocks will be between these two points. The fact of the matter is we cannot reduce risk (standard deviation) lower than that of the lowest return (Asset 1 in this case). Also, the portfolio will be on the line between assets 1 and 2 in the perfectly correlated case.

The expected return has nothing to do with the correlation. The expected return will depend on the weighted average (such as the 50% example shown in graph).

Consider here that the stocks are "moving together" (perfectly) as a result of their covariance. If this is true of all the assets in the portfolio you will have a great problem reducing risk. You need some assets moving in the opposite direction of the others (negative covariance) so that one stocks loss is offset by another's gain.

Perfect Negative Correlation

-Suppose $\rho = -1$. Then

$$\sigma_P^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\rho_{1,2}\sigma_1\sigma_2$$

$$= W_1^2\sigma_1^2 + W_2^2\sigma_2^2 - 2W_1W_2\sigma_1\sigma_2$$

$$= (W_1\sigma_1 - W_2\sigma_2)^2$$

$$\sigma_P = |W_1\sigma_1 - W_2\sigma_2|$$

-where |…| indicates the absolute value

Can we find a combination of weights which will reduce risk below that of the lowest standard deviation in the portfolio?

Example:

$\sigma_1 = 20\%$ $\sigma_2 = 30\%$ try $W_1 = .4$ $W_2 = .6$

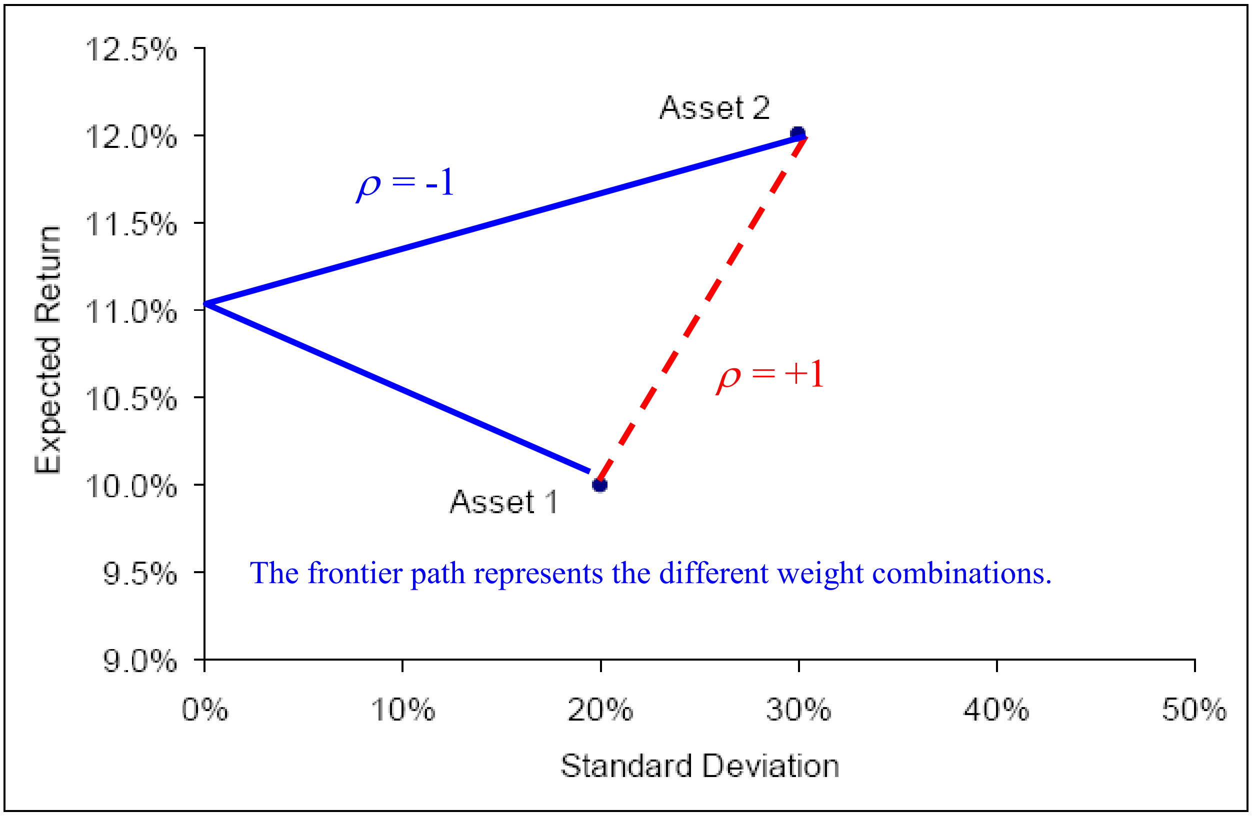
$\sigma_P = |.4*.2 - .6*.3| = 10\%$ YES, we have reduced risk

but we can also try

$\sigma_P = |.6*.2 - .4*.3| = |.12 - .12| = 0\%$

We have found a way to reduce risk entirely in the perfect negative correlation case. This shows the ability of a loss in one stock offset by a gain in another.

Perfect Negative Correlation



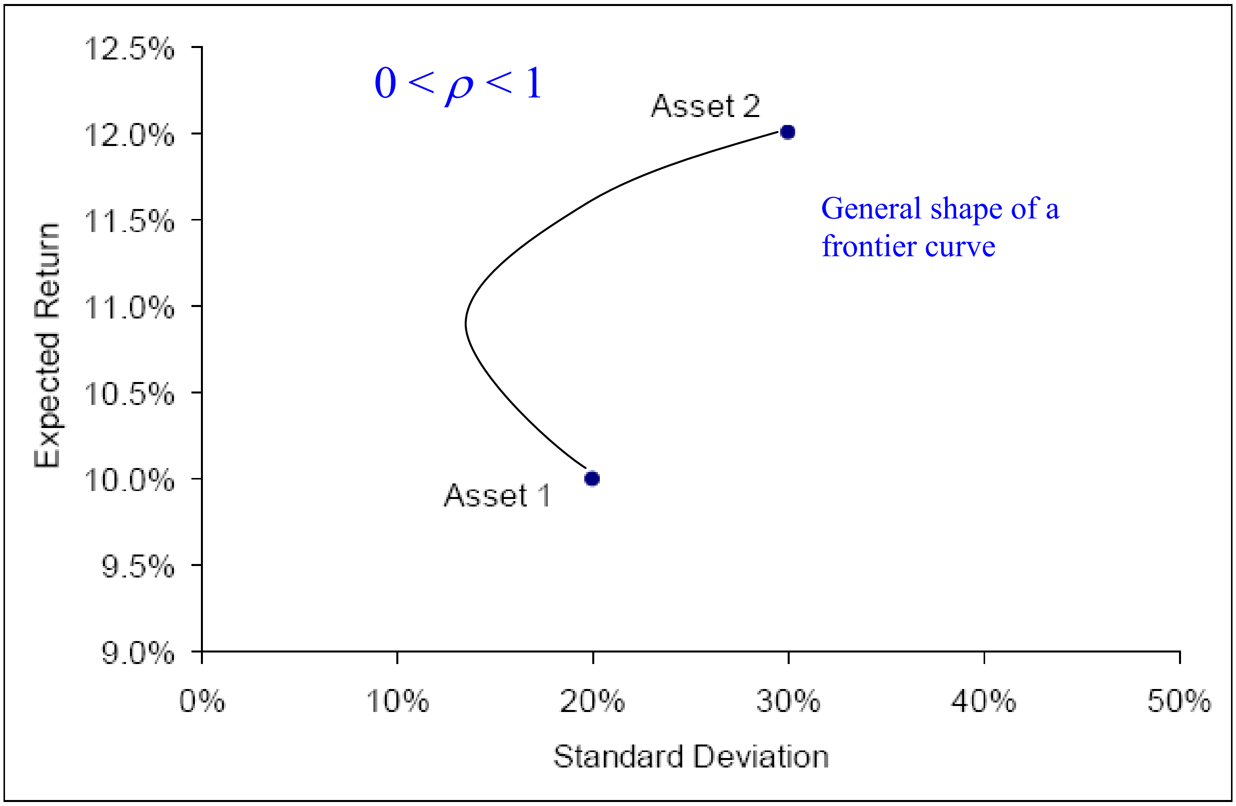
This shows the effect of reducing risk to 0 in the perfect negative correlation case. This is the idea of the investment frontier.

Other correlation

-It is rare that two assets have either perfect correlation or perfect negative correlation

-Most asset returns are positively (but not perfectly) correlated because they all are **affected by the same general economic conditions**

Other Correlation



General Case: $0 < \rho < 1$

In this case we will be able to reduce risk but not to zero. This is called the **General Case**, it represents the market influencing all stocks in the same direction depending on the state of the economy, either up or down.

The rules do not prevent perfect negative correlation but this is rare.

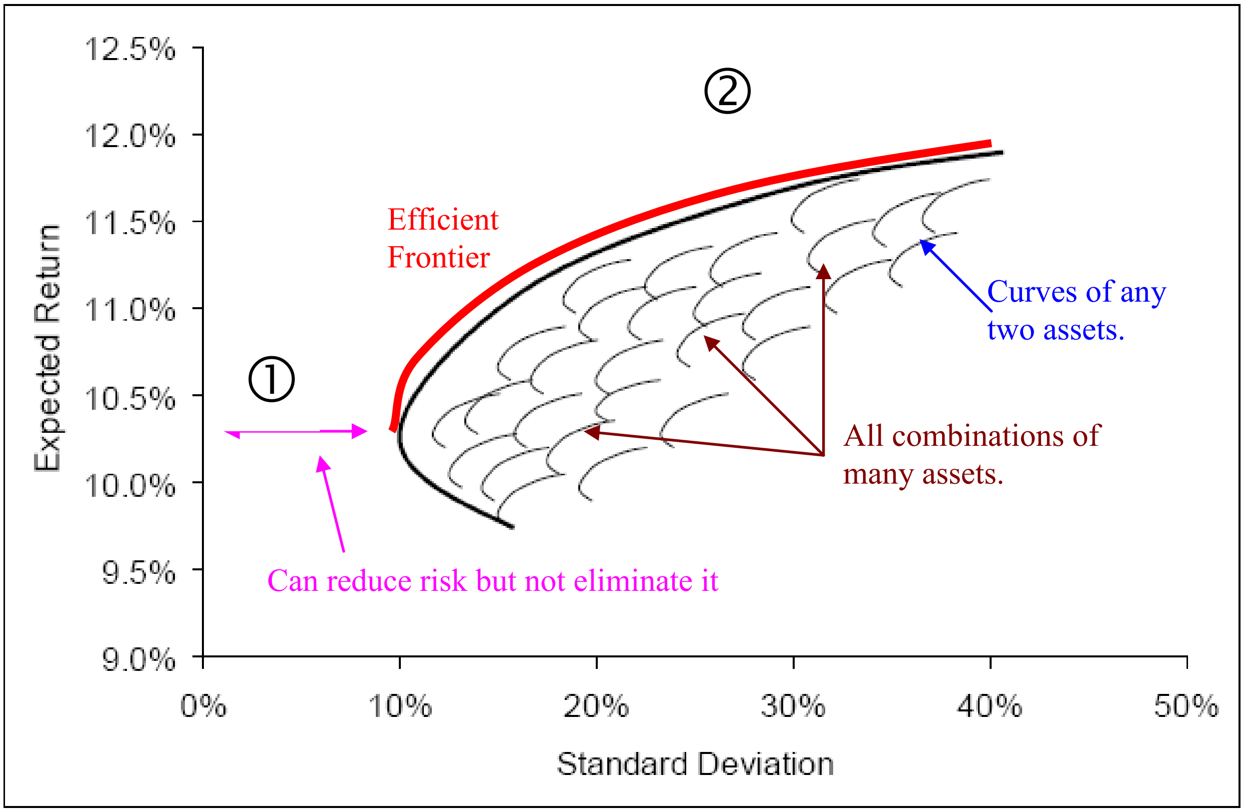
Risk in a portfolio context

-If your wealth is invested in a single asset, then asset 2 would be riskier than asset 1 because it has a higher standard deviation

-However, if you hold a portfolio of assets, the appropriate measure of risk is not the standard deviation, but the marginal contribution of the asset to the overall riskiness of your portfolio

-In this context, the riskiness of an asset can not be measured without reference to a benchmark portfolio

Portfolio choices with many risky assets



The portfolio we construct from many risky assets will be within the filled area.

1. Positive Correlation, usually the case, will not reach 0 risk.
2. Cannot get above (?)

Only a portfolio on the frontier is optimal. A Minimum Variance Frontier refers to minimizing standard deviation for a given expected return.

Now where to pick on the frontier? Want points above the point of minimum standard deviation but only above.

Note that we cannot say a portfolio with less risk and less return is better than a portfolio with more risk and a higher return.

Portfolio Choices with many Risky Assets

-The **minimum variance frontier** is the set of portfolios that minimizes the portfolio standard deviation for a given level of expected return

-The **efficient frontier** is the set of portfolios that maximizes the expected return for a given portfolio standard deviation

-The efficient frontier is the upward sloping portion of the minimum variance frontier

-**If investors prefer more to less** (other things equal, they prefer higher expected returns), and are **risk averse** (other things equal, they prefer a lower portfolio standard deviation), then all investors **should choose portfolios on the efficient frontier**.

The Limits to Diversification

-Diversification can reduce risk, but it cannot eliminate risk

	Unsystematic		*Systematic*
Total risk =	*Diversifiable*	+	*Non-diversifiable*
	Firm-specific		*Market-wide*

-Firm-specific risk can be eliminated from a portfolio by diversification. Market-wide risk cannot be eliminated by diversification

Say we have a couple of examples of RISK:
1. Bad Management (stealing)
2. Bad Economy

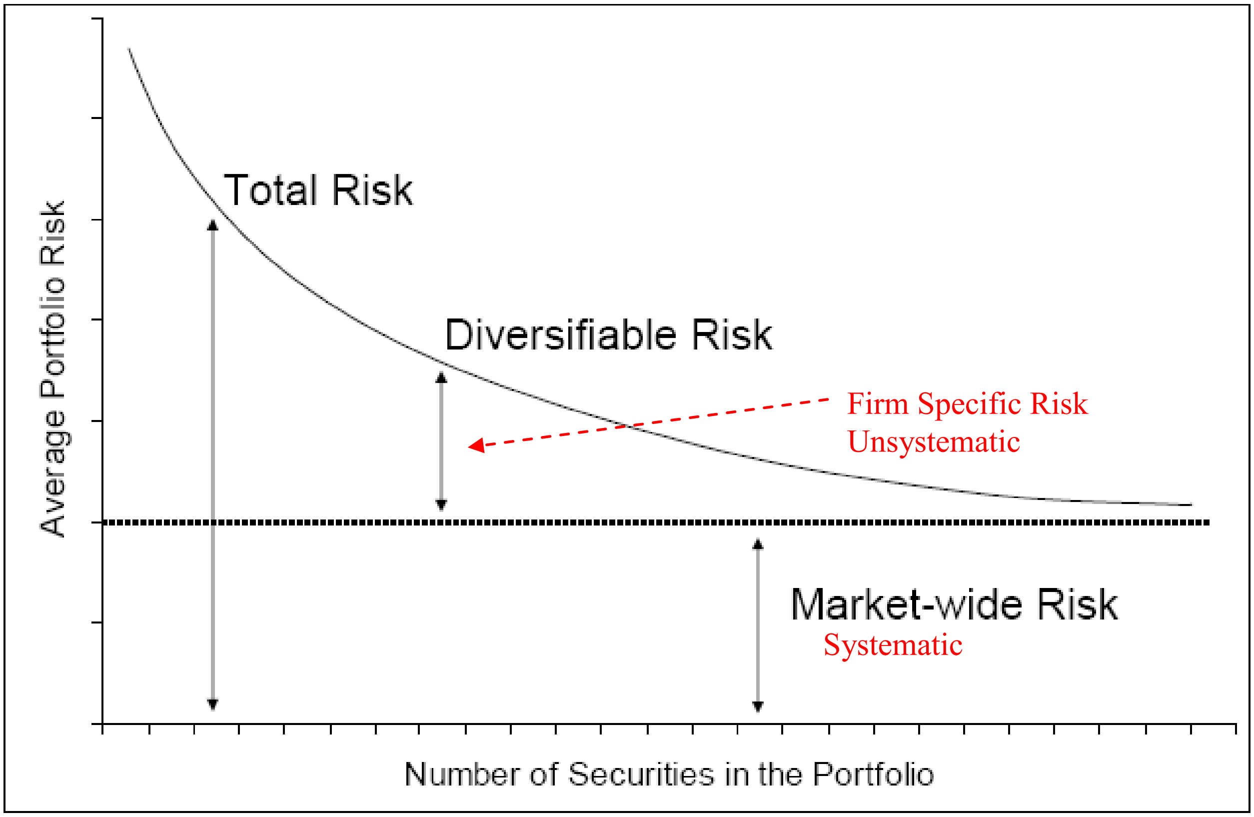
We can buy and construct a portfolio of 100 stocks to guard against the bad management risk. This is called Unsystematic Risk.

But we cannot buy and construct a portfolio which guards against the bad economy risk. This is called Symmetric Risk.

Firm Specific can be eliminated (Unsystematic).

“Systematic” because it’s from “the system” meaning the market.

The Limits to Diversification



The systematic risk is the difference between the point of minimum standard deviation on the frontier curve and the point of zero standard deviation.

As we increase the number of securities in the portfolio we can eliminate the firm specific risk (unsystematic) but cannot eliminate the market-wide systematic risk. Now, how many stocks (or other assets) are enough to optimize a portfolio?

Diversification – How many Stocks?

- Conventional wisdom:

• Evans and Archer (1968): about 10 stocks is usually sufficient

• Statman (1987): more like at least 30-40

- Often, most **diversification** will have incurred by the time your 20th stock is purchased. This is typically enough to eliminate the unsystematic risk.

Practice Question

10.2 10.5 10.11 10.14 10.15 10.16

Return & Risk: The Capital Asset Pricing Model (CAPM)

The purpose of Chapter 10 is to more formally explain the relationship between return and risk that was first introduced in Chapter 9. This relationship is the capital asset pricing model (CAPM). Chapter 10 discusses how individuals may view a security's variance as an appropriate measure of risk. For a portfolio of assets, however, investors should view a security's contribution to the risk of the portfolio as the appropriate measure of risk. This contribution is indicated by the security's beta, which is a measure of a security's covariance risk with respect to the market portfolio. The chapter provides the necessary mathematics to calculate the expected return and variance of a portfolio containing many risky assets.

The major concepts discussed in Chapter 10 are outlined below.

- Individual securities
    - returns
    - variances
    - covariances
- Portfolios
    - returns
    - variances
    - covariances
- Efficient set for two assets
- Efficient set for many assets
    - variance
    - diversification
- Riskless borrowing and lending
    - optimal portfolio
    - capital market line
- Market equilibrium
    - homogeneous expectations
    - beta
    - characteristic line
- Capital Asset Pricing Model
    - security market line

Chapter Summary

This chapter sets forth the fundamentals of modern portfolio theory. Our basic points are these:

1. This chapter shows us how to calculate the expected return and variance for individual securities, and the covariance and correlation for pairs of securities. Given these statistics, the expected return and variance for a portfolio of two securities *A* and *B* can be written as

$$\text{Expected return on portfolio} = X_A\overline{R}_A + X_B\overline{R}_B$$
$$\text{Var(portfolio)} = X_A^2\sigma_A^2 + 2X_AX_B\sigma_{AB} + X_B^2\sigma_B^2$$

2. In our notation, *X* stands for the proportion of a security in one's portfolio. By varying *X*, one can trace out the efficient set of portfolios. We graphed the efficient set for the two-asset case as a curve, pointing out that the degree of curvature or bend in the graph reflects the diversification effect: The lower the correlation between the two securities, the greater the bend. The same general shape of the efficient set holds in a world of many assets.
3. Just as the formula for variance in the two-asset case is computed from a 2×2 matrix, the variance formula is computed from an $N \times N$ matrix in the *N*-asset case. We show that, with a large number of assets, there are many more covariance terms than variance terms in the matrix. In fact, the variance terms are effectively diversified away in a large portfolio but the covariance terms are not. Thus, a diversified portfolio can only eliminate some, but not all, of the risk of the individual securities.
4. The efficient set of risky assets can be combined with riskless borrowing and lending. In this case, a rational investor will always choose to hold the portfolio of risky securities represented by point A in Figure 10.9. Then he can either borrow or lend at the riskless rate to achieve any desired point on line *II* in the figure.
5. The contribution of a security to the risk of a large, well-diversified portfolio is proportional to the covariance of the security's return with the market's return. This contribution, when standardized, is called the beta. The beta of a security can also be interpreted as the responsiveness of a security's return to that of the market.
6. The CAPM states that

$$\overline{R} = R_F + \beta(\overline{R}_M - R_F)$$

   In other words, the expected return on a security is positively (and linearly) related to the security's beta.