The Capital Asset Pricing Model (CAPM)

Still pursuing the question “what is the contribution of a stock added to a portfolio to the risk of the portfolio”? Can we do better by adding the risk free asset?

Portfolios with one risky asset and one risk-free asset

-A risk-free asset has zero variance and zero covariance with any other asset, it is a specific kind of asset with standard deviation of 0.

-Thus, a portfolio with one risky asset and one risk-free asset has an expected return of

$E(R_P) = W_1E(R_1) + W_2R_f$ (where R_f is the Risk Free component, $E(R_f) = R_f$) and a standard deviation of

$$\sigma_P = W_1\sigma_1$$

Will adding a risk free component help us to find a more efficient portfolio? First we must examine the characteristics of the Risk and Risk-Free portfolios.

$E(R_f)$ = constant (no risk) = R_f. Risk free asset has $\sigma^2{}_p = 0$. Also, the covariance of a random variable and a constant is 0, the constant does not change with the random variable. Cov(X, a) = (2*E(X))(7–7)*.2 + … this will continue for all terms. So we have:

$$\sigma^2{}_p = W_1{}^2\sigma_1{}^2 + \cancel{W_2{}^2}\sigma_2{}^2 + 2\cancel{W_1}W_2\sigma_{1,2} \quad \text{(for a two item portfolio)}$$

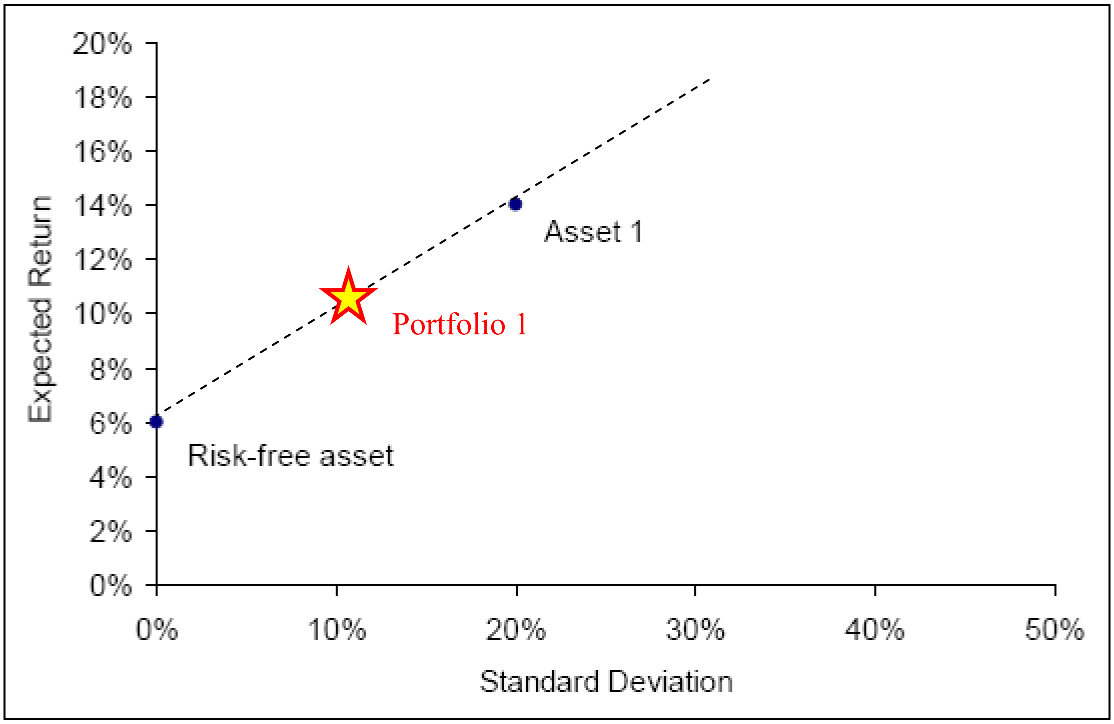
The result is $\sigma^2{}_p = W_1{}^2\sigma_1{}^2$ and $\sigma_p = W_1\sigma_1$.

Example

-Consider the following information. The expected return on the risky asset is 14%, and its standard deviation is 20%. The return on a risk-free asset is 6%

-Calculate the **expected return** and **standard deviation** for the following portfolios:

- Portfolio with **60% in the risky asset** and **40% in the riskless asset**



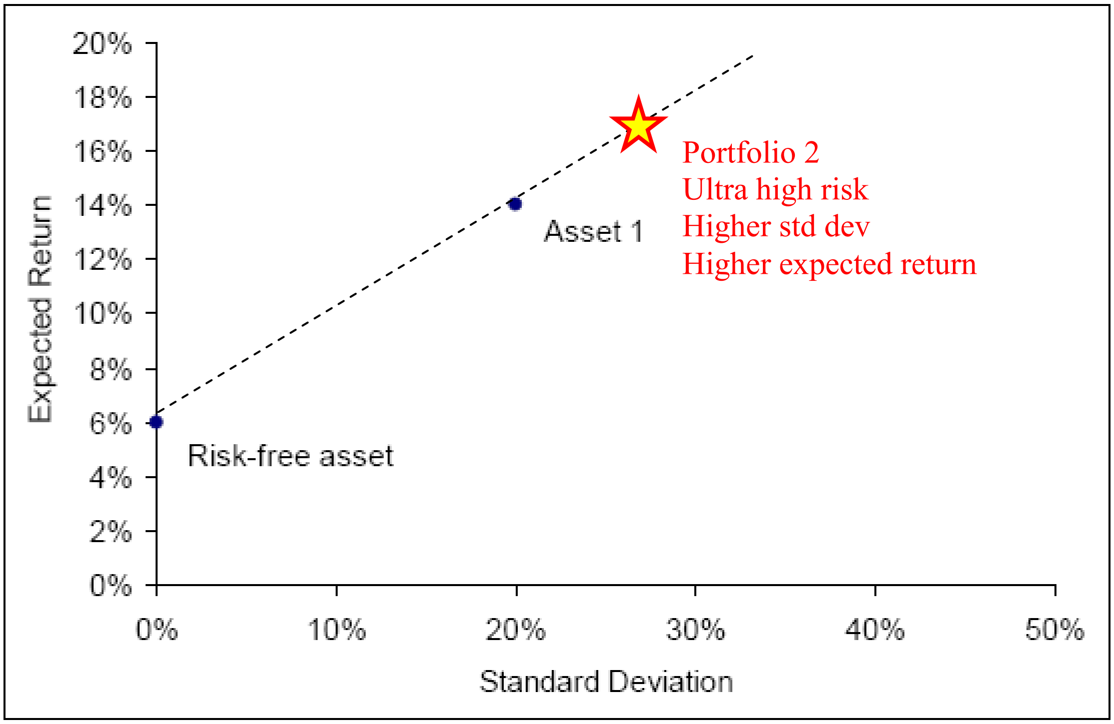
$E(R_p) = W_1E(R_1) + W_2R_f = .6*.14 + .4*.06 = 10.8\%$ $\sigma_p = W_1*\sigma_1 = .6*.2=12\%$

We see it is on the line because it is just derived from proportions of the same info, the point only moves along the line. The possible portfolios which we can construct from these two assets are on a straight line between the risk free asset and asset 1. This is simple to see when we consider that what constitutes a portfolio is a distribution of weights among the assets. If there are only two assets the weights can only be manipulated to move the portfolio point along the line.

Example

- Portfolio where an investor **borrows** 20% of his wealth and invests everything in the risky asset:

Ex. We have $1,000,000, borrow 200,000, invest it all in Asset 1.



What are the weights here? 120% goes into risky asset meaning we must borrow $200,000 which we add to our investment. Now W1 and W2 will not be positive, the percentage owed to the bank will be negative. The portfolio which includes borrowing is in a riskier position, plots above the riskiest asset. Key is to be making more on the funds you borrow than the cost of borrowing!

$E(R_p) = 1.2^*.14 + (-.2)^*.06 = 15.6\%$ (here we are assuming that we can borrow at 6%)

$\sigma_p = 1.2^*.20 = 24\%$

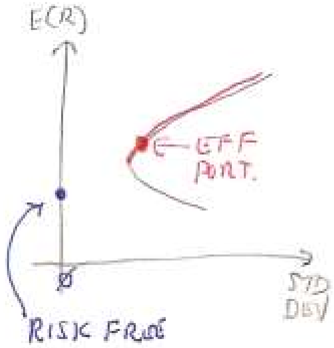
Note: THE WEIGHTS ALWAYS SUM TO ONE !!!

Note: WHEN YOU BORROW TO INVEST W1 AND W2 WILL NOT BOTH BE POSITIVE, THE PERCENTAGE OWED TO THE BANK WILL BE NEGATIVE. BUT THE TWO WEIGHTS WILL STILL SUM TO ONE.

Efficient portfolios with borrowing and lending

-By combining risky securities we were able to generate and efficient portfolio frontier

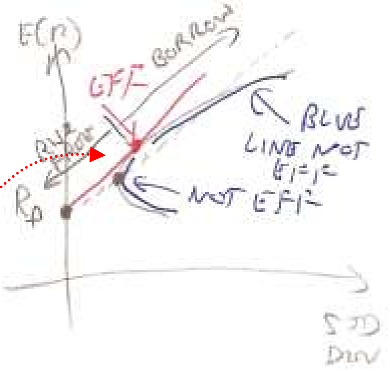
-If we assume the existence of risk-free assets, could we obtain even better risk/return combinations? A "better" portfolio is one in which you keep the same std dev but obtain a higher expected return OR keep the same expected return and lower the standard deviation.



In the presence of risk free assets can we reduce risk (standard deviation)?

Better portfolio is defined as higher return with same of lower risk.

We know from previous pages that we can construct a portfolio on the efficient frontier **and** anywhere on the line formed by that portfolio and the risk free investment.

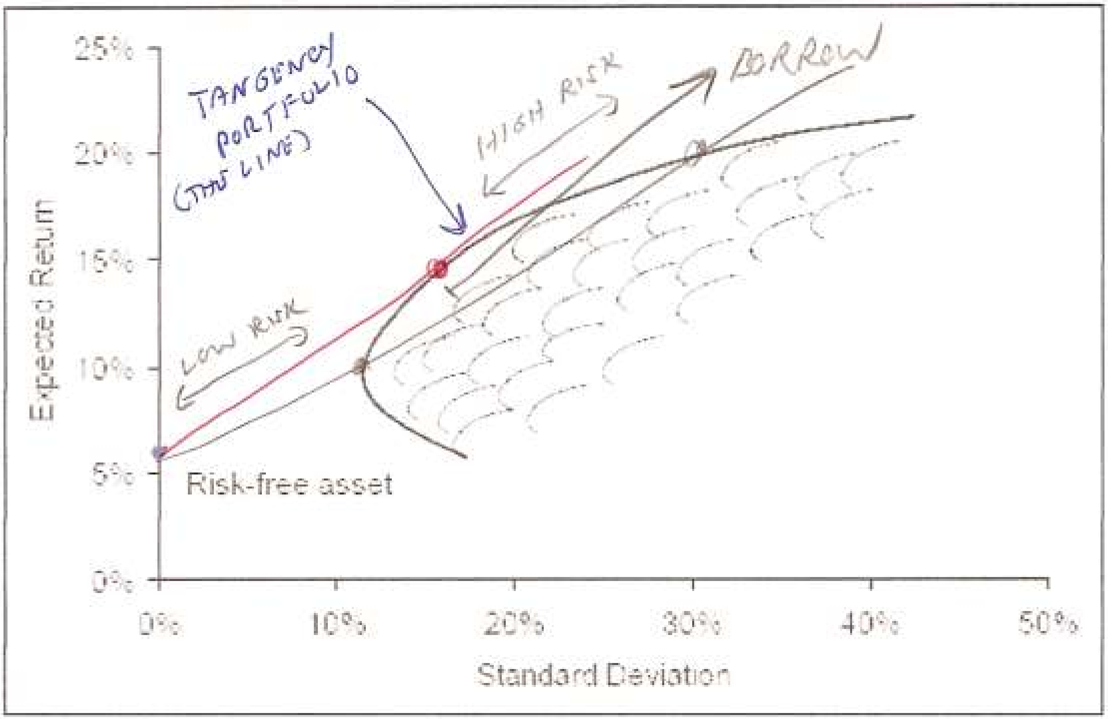


Which portfolios are efficient? **The line is efficient if we select it properly**. All of the portfolios which were efficient before (when we did not have a risk free component) are now no longer efficient with the exception of one portfolio, the point where the line from the risk free asset intersects the tangent of the frontier curve. THE LINE MUST INTERSECT THE TANGENT OF THE POINT ON THE EFFICIENT FRONTIER.

Also, the entire blue line is **NOT efficient** in the presence of risk free investments. We can choose weights to put us anywhere on the efficient line. We now have a **NEW EFFICIENT FRONTIER** in the presence of **RISK FREE ASSETS, it is the line which connects the risk free asset and the tangent of the "efficient" frontier curve.**. Only portfolios on this line are efficient in the presence of risk free assets. We have found that "we can do better than other risky assets". But moving toward risk-free lowers the expected return.

This new efficient frontier is called the CAPITAL MARKET LINE (CML). The portfolio which remains efficient, the point where the CML intersects the efficiency frontier, is called the TANGENCY PORTFOLIO.

Portfolios with many risky assets and one risk-free asset



In the absence of a risk free rate the investor decides on the level of risk he can tolerate and then selects the appropriate point on the efficient frontier. Above we see two points on the efficient frontier which represent two different weight combinations designed to give two different risk/return options. But the tangency portfolio is efficient.

Any two points on the efficient frontier are equally good investments, they just have different risk levels.

It doesn't matter what your risk aversion level is, all investors in the market will eventually hold the same mix of risky assets which is the **Tangency Portfolio**. Everyone in the market will do the same, first find the tangency portfolio, then, based on the risk aversion level decide how to combine tangency portfolio with the risk free assets. This is the **separation principle**.

Separation Principle: model says everyone in market goes to the same tangency portfolio and works up or down the line based on their risk tolerance. First the tangency portfolio, we separate the risk aversion level from the mix of the risky assets. Regardless of the risk aversion level everyone will hold the same mix of risky assets. The only difference will be how to combine them with the risk-free assets.

All demand must equal all supply in the market so the proportion of stocks in the tangency portfolio equals the proportion of the firms (the value of the firms) in the entire market. For example, if everyone wants their proportion of Microsoft stock to be 1% then if we calculate the value of Microsoft and divide it by the value in the entire market it should equal 1%. Everyone will want their money invested in this proportion exactly, so the money invested in Microsoft will be exactly 1% of all the money invested in the market.

This is why the tangency portfolio is called the **Market Portfolio**, because the proportions of the stocks in that unique mix of the tangency portfolio must be the proportion of the value of the firm in the market.

So the weight of each stock in the tangency portfolio will be the market value of the stock divided by the sum of all value of all firms in the market.

-With one risk-free asset and many risky assets, efficient portfolios lie on a straight line from the risk-free rate that is tangent to the efficient frontier of risky assets

-The line from the risk-free rate that is tangent to the efficient frontier is sometimes called the **capital market line**. The unique portfolio of risky assets that is also on this line is called the **tangency portfolio**.

Separation Principle: Suppose all investors have homogenous expectations with respect to expected returns, standard deviations of returns, and correlations between returns.

-Then all investors, regardless of their risk preferences, will form optimal portfolios by combining only two assets: the risk-free asset and the tangency portfolio. - i.e., the investors separate their risk aversion from their choice of the stocks portfolio

The Market Portfolio: -The separation principle implies that all investors will hold the same portfolio of risky assets.
-In equilibrium, **supply must equal demand** for all assets
-Thus, in equilibrium, the tangency portfolio must be the market portfolio with portfolio weights given by

$$W_i = \frac{\text{Market value of asset i}}{\text{Total market value of assets}}$$

	3 Risky Assets (stocks)			
Risk Free	X	Y	Z	
Asset	20%	50%	30%	Three Investors / Firms
800	.2*200=40	.5*200=100	.3*200=60	John $1,000 (800 in R_f)
-1000	.2*3000=600	.5*3000=1500	.3*3000=900	Jane $2,000 (borrows 1000)
0	.2*700=140	.5*700=350	.3*700=210	Sharon $700 (all in Tangent Portfolio)
Total:	**780**	**1950**	**1170**	Market value of firms

$3900 = Value of all three firms in market (stocks, not including risk free)

We see that …

780/3900=**20%** 1950/3900=**50%** 1170/3900=**30%** t is built this way due to tangency portfolic

Note that "all in tangency portfolio" means zero in risk free!

The proportion of each stock in the tangency portfolio must be the same proportion of the value of each firm in the entire market.

At the beginning of the class we discussed needing a portfolio which would be a good candidate for a benchmark to evaluate the contribution of any other stock to that portfolio. We see now that the market portfolio is a good candidate for that portfolio. Because based on what we have seen so far, all the investors in the market will want to hold the same portfolio. We will use this portfolio (market portfolio) as a benchmark to evaluate the relevant risk of the individual stocks. We will measure what is the contribution of each stock to the risk of that portfolio, and that contribution will ultimately be converted to the cost of capital.

The Market Portfolio

-Earlier we said that risk must be measured with reference to a **benchmark portfolio**

-If, in equilibrium, all investors are holding the market portfolio of risky assets, it is natural to measure risk with reference to this portfolio

The market portfolio is a good benchmark portfolio.

The Capital Asset Pricing Model

-The appropriate risk premium on an asset is determined by its contribution to the risk of investors' overall portfolios

-In equilibrium, any two assets with the same marginal contribution to the risk of the market portfolio must have the same expected return

-The **Capital Asset Pricing Model**, or **CAPM**, puts a specific functional form on this relationship

We have a Market Portfolio and we add a stock to it increasing the risk of the market portfolio. To add this stock we require a higher rate of return to compensate for the increased risk the stock brings to the portfolio. The opposite would be true of a stock added which reduced the risk of the portfolio. To calculate the amount of compensation we need to add a stock we ca use the **CAPM** equation. CAPM converts the risk of the stock to a proper discount rate. We will estimate the cost of capital based on the contribution of risk the asset brings to the portfolio. We have a model which allows us to do this…

The Capital Asset Pricing Model

-What is the marginal contribution of a single asset to the risk of the market portfolio? Based on this measure the model will derive the cost of capital. In other words, it converts the contribution of the stock to the riskiness of the portfolio to the discount rate. This is our ultimate goal, finding a proper discount rate.

First we will look step-by-step at the intuitive ideas behind the model.

Consider a thought experiment:

- Suppose all investors solve the portfolio optimization problem and all decide to hold the market portfolio
- Suppose that a new stock is listed on the exchange
- How should the stock be priced on the market? i.e., what should be the required return on that stock? (should depend on the risk level)

-What is the variance of a portfolio (P) that has a weight of W in the new stock (i) and (1-W) in the previous market portfolio (M)? [W is the new investment, W-1 is the old investment, R_M is market Portfolio Return, R_i is the return of the new stock] What is the variance of the new portfolio? Will it be higher or lower than the old portfolio?

$$\underbrace{\mathrm{Var}(R_P)}_{\text{Variance with the new stock}} = W^2\mathrm{Var}(R_i) + \underbrace{(1-W)^2\mathrm{Var}(R_M)}_{\text{Variance before adding the new stock}} + 2W(1-W)\mathrm{Cov}(R_i,R_M)$$

-Suppose we decide to invest a small fraction of our wealth in the new asset, say 1%
-Then, the variance of the portfolio (including the new stock) is

$$\begin{aligned}
\mathrm{Var}(R_p) &= .01^2{*}\mathrm{Var}(R_I) + .99^2{*}\mathrm{Var}(R_M) + 2{*}.01{*}.99{*}\mathrm{Cov}(R_I, R_M) \\
&= \cancel{.0001{*}\mathrm{Var}(R_I)} + .9801{*}\mathrm{Var}(R_M) + .0198{*}\mathrm{Cov}(R_I, R_M) \\
&\approx 0 + .98{*}\mathrm{Var}(R_M) + .02{*}\mathrm{Cov}(R_I, R_M)
\end{aligned}$$

When we add a new stock it effects the variance of the portfolio through it's **covariance** with the rest of the portfolio and **NOT** through it's variance.

-Small positions add to the variance of the market portfolio not through their own variance but through their **covariance** with the market.

-All individual positions are small if one is holding the market portfolio.

-**Covariance with the market is the relevant measure of risk for individual assets**.

The Capital Asset Pricing Model

So lets say we have added the stock, what happens to the variance of the portfolio? Does adding the stock increase or decrease the variance of the portfolio?

-If

$$Var(R_P) \approx 0.98 * Var(R_M) + 0.02 * Cov(R_i, R_M)$$

then **the variance will increase whenever** $Cov(R_i, R_M) > Var(R_M)$

Or, **the variance will increase whenever** $\dfrac{Cov(R_i, R_M)}{Var(R_M)} > 1$

If this ratio is greater than 1 then the risk will increase. We have added a stock and now must ask, is the portfolio variance higher or lower then the market portfolio?

If $Var(R_M) = Cov(R_I, R_M)$ then no change in the variance of the portfolio, $Var(R_P)$ (no effect on the risk of the portfolio).

Say $Var(R_M) = 20\%$ and $Cov(RI, RM) = 10\%$ then we will see an decrease in the variance of the portfolio.

If $Cov(R_I, R_M)$ were negative this would reduce the variance, but assets like this are hard to find.

If $Cov(R_i, R_M) > Var(R_i)$ then adding the stock will increase the risk of the portfolio.

If $Cov(R_i, R_M) < Var(R_i)$ then adding the stock will decrease the risk of the portfolio.

Generally we are always going to see an increase in the portfolio variance, $Var(R_P)$.

If adding a new stock to the market portfolio increases the risk of the portfolio then the expected return of the portfolio should also increase. This is saying:

$$\frac{COV(R_I, R_M)}{VAR(R_M)} > 1 \quad \Rightarrow \quad E(R_I) > E(R_M)$$

(see next page)

The Capital Asset Pricing Model

-If adding a single stock to the market portfolio increases the variance of the portfolio return, then it should also increase the expected return on the portfolio

-The portfolio return will be higher when

$$E(R_P) = WE(R_i) + (1-W)E(R_M) > E(R_M)$$

$$\text{i.e., when} \quad E(R_i) > E(R_M)$$

-Thus:

$$\boxed{\frac{\mathrm{Cov}(R_i, R_M)}{\mathrm{Var}(R_M)} > 1 \Rightarrow E(R_i) > E(R_M)}$$

In terms of expected return

$$E(R_P) = WE(R_I) + (1\text{-}W)E(R_M) = .01E(R_I) + .99E(R_M)$$

Say we have $E(R_M) = 30\%$ and $E(R_I) = 30\%$, then there will be no change in the expected return of the portfolio, $E(R_P)$. But say we have $E(R_I) = 40\%$ and $E(R_M) = 30\%$, then $E(R_P)$ will be higher. Likewise, if we have $E(R_I) = 20\%$ and $E(R_M) = 30\%$, then $E(R_P)$ will be lower.

$E(R_P)$ will INCREASE when $E(R_I) > E(R_M)$.

$$\beta_i = \frac{\mathrm{COV}(R_i, R_M)}{\mathrm{VAR}(R_M)}$$

Beta measures the **Relevant Risk**, referred to as "the beta of the stock".

-Beta is the risk measure relevant for individual securities in a well diversified portfolio (like the market). If beta is high the cost of capital will be high. Each stock has its own beta. Beta is the measure of the relevant risk of a stock. Since beta is a function of the covariance of the stock added to the portfolio we know that the relevant risk is systematic (market risk).

Beta measures the risk for a security that captures only the systematic, non-diversifiable, component of the security's risk.

The Capital Asset Pricing Model

-We saw that

$$\beta_i > 1 \Rightarrow E(R_i) > E(R_M)$$

-The general relationship between β_i and $E(R_i)$ is given by the following formula known as the CAPM formula (R_f is the return of the risk free asset):

$$\boxed{\text{CAPM Formula:} \quad E(R_i) - R_f = \beta_i(E(R_M) - R_f)}$$

This is saying:
The expected return of the new asset – return of the risk free asset = beta * (the difference of the expected return of the market portfolio – the return of the risk free asset).
[$(E(R_M) - R_f)$ is the market risk premium] CAPM is a linear function of beta,

-The CAPM predicts that the expected return on any asset is linearly related to its beta

$$\boxed{E(R_i) = R_f + \beta_i(E(R_M) - R_f)}$$

-This relation is interpreted as

$$E(R_i) = (\text{Price of time}) + (\text{Risk premium}) =$$

$$= (\text{Price of time}) + (\text{Amount of risk})^*(\text{Price of risk})$$

Any asset follows this rule. Beta (amount of risk) is different for each firm. Beta will be the amount of risk. The Price of Risk will be the same for everyone (all investors?).

R_f is always positive due to the time value of money, for this reason the risk free rate is called "**The Price Of Money**" or "**The Price Of Time**".

CAPM intuition

-One way to interpret beta is as a measure of responsiveness. For example, if the market's excess return is 10%, then an excess return of a stock with a beta of 1.5 is expected to be 15%

- Likewise, if the market's excess return is -3%, then an excess return of a stock with a beta of 1.5 is expected to be -4.5%. Always the same proportion due to linear relation.

-Most firms have positive betas

-What is the beta of the market portfolio?

Linear relationship, look at the risk premium, the excess return of the firm should be proportional due to the linear relation. So in a way we can say how each firm responds to the market return.

β represents a proportion of the firms return based on the market return. Same proportion because linear relationship.

β can also measure a portfolio's response or responsiveness.

Most firms have positive betas because most firms are positively correlated to the market.

We can also consider the beta of an entire portfolio (as opposed to just a single stock) because any portfolio that has some expected return and standard deviation will also have some covariance with the market. Any asset that will have some covariance with the market will have it's own beta. So now we know we can measure beta not only for a single asset but for a collection of assets.

Now what will be the beta of the market portfolio?

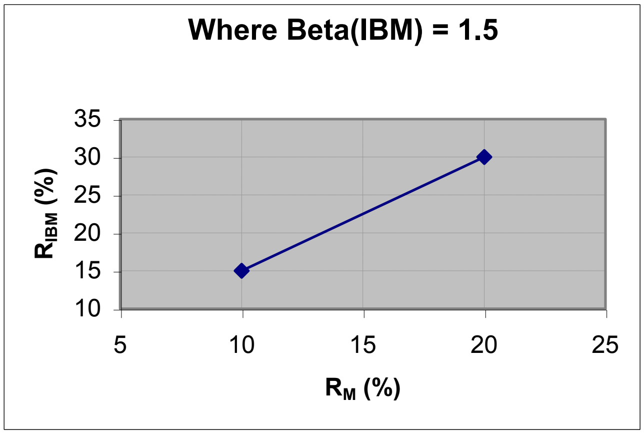
$\text{Market } \beta = 1$ because it is all the stocks in the market.
Average of all β's is 1 because this is how the market return is determined. Say the market return is 10%, how did we find this return? By taking the average of all returns of all stocks. So the average of all returns of all stocks in the market will be 10% and the average beta of all stocks will be 1. So the average of all betas in the market is one.

The Security Market Line

-The **security market line** is the graph of the linear CAPM relation

$$E(R_i) = R_f + \beta_i (E(R_M) - R_f)$$

-The CAPM predicts that all assets (and all portfolios of assets) lie on the security market line. Thus, **the CAPM relates the required rate of return for any security with the risk for that security, as measured by beta.**

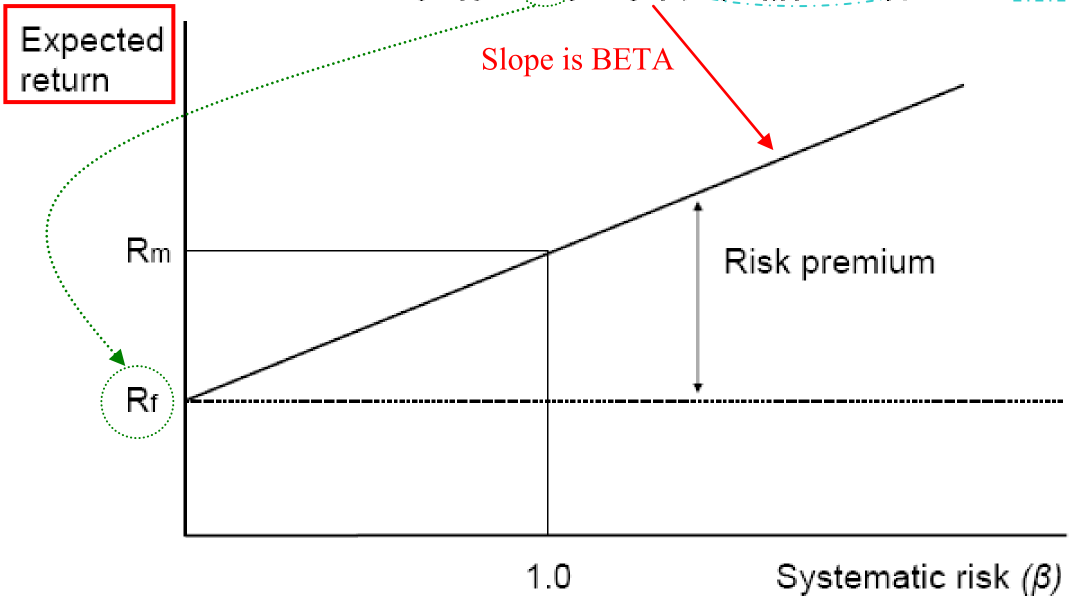


Security Market Line = CAPM Line (but this is **not** the same as the CML line).

The actual returns will not always be on the line. This is a model predicting an ideal result. Actual measurements will have imperfections. Trying to find beta through measurements will be difficult because the points will not fall on straight lines (next page). Also, some of the model assumptions will not hold. The theory may consider all relevant facts. Typically we will find that the data does not correspond perfectly to what the model says it should be.

$$E(R_i) = R_f + \beta_i (E(R_M) - R_f)$$

Will be positive because $E(R_M)$ compensates for risk.

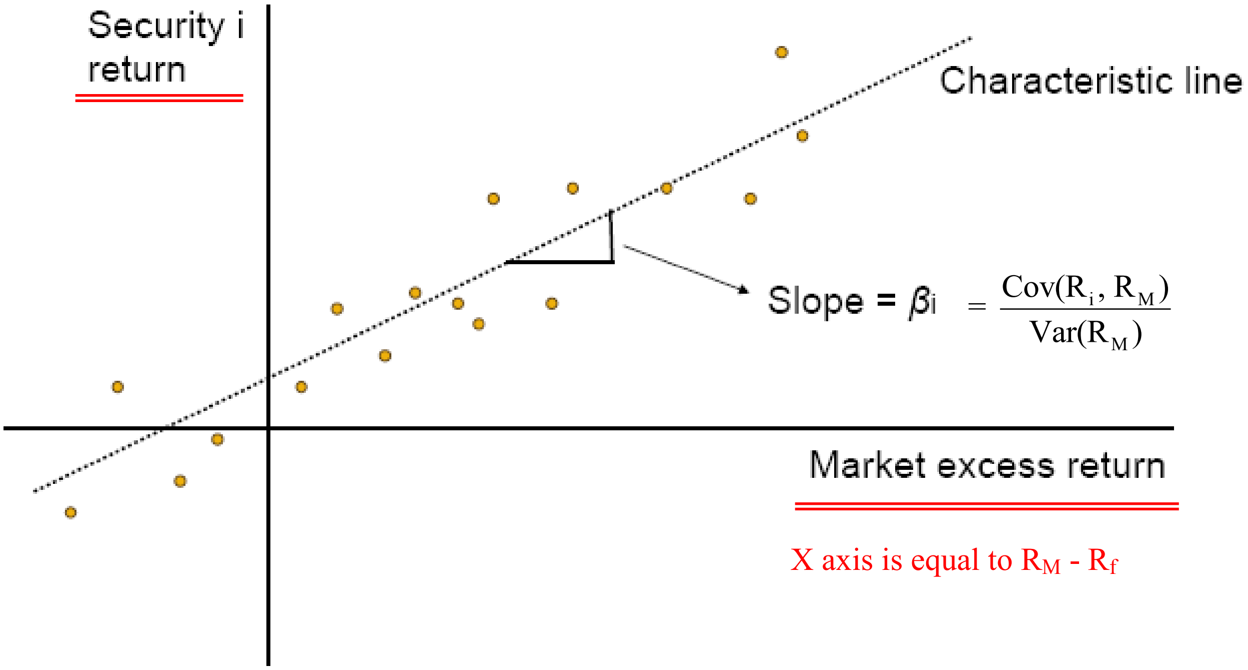


if β=0 the stock is not risky, $E(R) = R_f$
if β=1 then E(R) = Market Return
if β>1 then E(R) > Market Return

$E(R_M)$ is always higher than the risk free rate, R_f. So the difference, $E(R_M) - R_f$, is always positive (on average it will always be positive, a random variable can return a negative expected value)..

Estimating Beta

$$E(R_i) = R_f + \beta_i\left(E(R_M) - R_f\right)$$

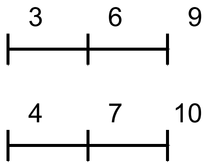


X axis is equal to R_M - R_f

Each point represents a different return on a different day (say we tracked it for the last month for instance). Find the line using regression (least squares estimate in this case). (Note the axis, we are plotting the return of the security against the market excess return). The X axis value equals the Return of the Market minus the Risk Free return (constant).

$$y = a + bx \qquad \hat{b} = \frac{\text{Cov}(X, Y)}{\text{Var}(X)} = \beta \qquad \beta \approx \frac{R_i - R_f}{R_M - R_f}$$

Var(X – a) = Var(X) where a is a constant.



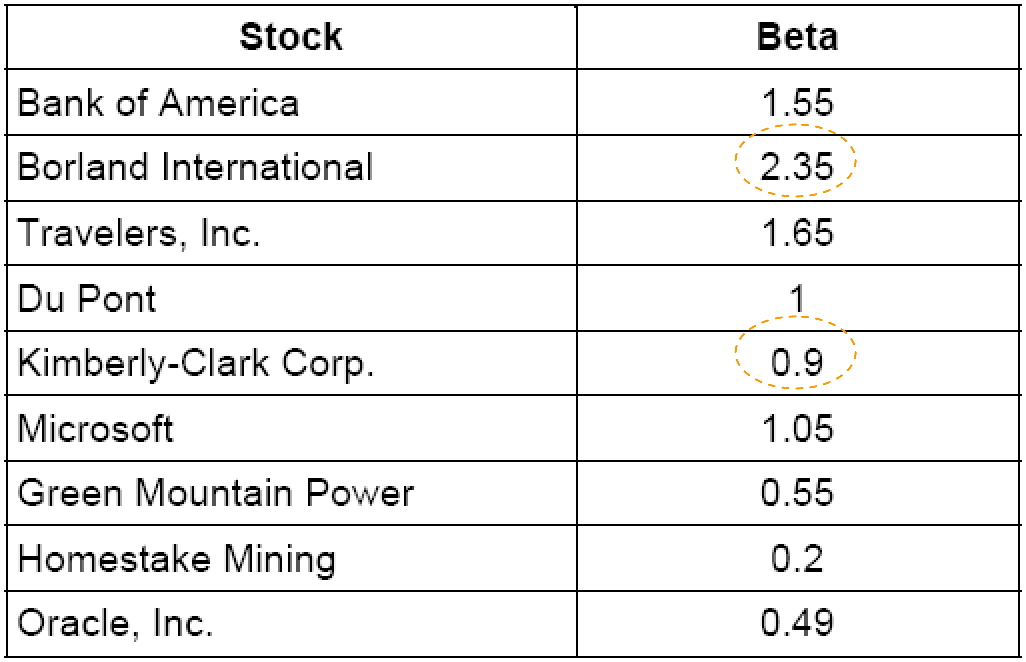
Movement is the same, no variation.
same variance

Also, Cov(x, y-a) = Cov(x,y) – Cov(x,a) = Cov(x,y) because Cov(x,a) = 0, a is a constant and does not change with x.

So the constant terms in the beta estimate eq can be eliminated (R_f).

Estimating beta for selected stocks

(real numbers from analysis)



Stock	Beta
Bank of America	1.55
Borland International	2.35
Travelers, Inc.	1.65
Du Pont	1
Kimberly-Clark Corp.	0.9
Microsoft	1.05
Green Mountain Power	0.55
Homestake Mining	0.2
Oracle, Inc.	0.49

Is KC variation greater than Borland variation?

$$\beta_i = \frac{\text{COV}(R_i, R_M)}{\text{VAR}(R_M)}$$

We cannot make an assumption about the variance of R_i. The reason is because variance is NOT the relevant risk!

Covariance ratio (beta) is the relevant risk! Lower beta means the firm has lower covariance with the market but does not mean is has lower variance.

Based on this table we can make statements about the firms covariance with the market but not about variance. The variance, or standard deviation, is the relevant risk of a single stock. But a portfolio can only compare the covariance between a stock and itself.

Example

-The current risk-free rate is 5%, the expected return on the market portfolio is 8%, and security A has a beta of 1.2.

According to CAPM, what is the expected return on security A?

How do we convert β to the cost of capital? Use the CAPM model:

$$E(R_A) = R_f + Beta*[E(R_M) - R_f] = 5\% + 1.2*(8\% - 5\%) = 8.6\%$$

So we have added a stock with relevant risk of 1.2 and found the required return to be 8.6%.

If the expected return on stock B is 11.3%, what is the beta of stock B?

$E(R_B) = 11.3\% = 5\% + \beta_B*(8\% - 5\%)$ → $\beta_B = 2.1$ this says that stock B is more risky.
We see that stock B is more risky (higher beta).
Now we can convert to Cost of Capital based on CAPM equation:

CAPM Formula: $E(R_i) = R_f + \beta_i(E(R_M) - R_f)$

Concept check

EXAM

These are the types of concept questions to expect on the exam.
COULD BE THESE SAME QUESTIONS

True or False?

- Stocks with a beta of zero offer an expected rate of return zero
FALSE, just look at the CAPM equation, the return would be the risk free return, R_f.

- The CAPM implies that investors require a higher return to hold ~~highly volatile securities~~ stocks with higher betas, stocks that have higher covariance with the market. **FALSE**

- If a stock has a negative beta, then the stock return is negatively related to the Market Return. **TRUE**. We can see this in the CAPM equation. In such a case beta has negative covariance to the market. Beta measures how the stock reacts to the market.

Not that negative beta directly implies negative covariance (negative correlation).

Assumptions behind CAPM

-There are many investors none of whom has market power.
Saying that no one investor can influence the market and that all are price takers. Not true but close enough (pretty close to true).

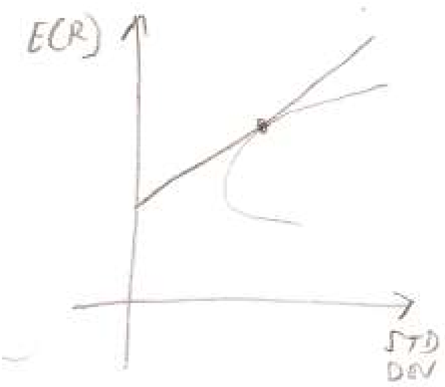
-All investors have an investment horizon of one "period".
We have built this model based on the idea that each investor is looking for the performance in the coming 1 period. The model is based on one period. This is not a reliable assumption, it's a problem but necessary for the model to work.

-Investors have homogeneous beliefs.
Says investors believe in standard deviation and proportional expected return. But investors with more and/or better information will draw different conclusions so this assumption is not true, not all investors in the market are drawing the same conclusions.

-All investors are risk-averse.
To see this we only have to consider that the risk premium is twice the risk free rate, 8% versus 4%. This is a good assumption.

-Capital markets are perfect. This means no transaction costs, no taxes and no restrictions on short selling.
This is not true and impacts the model but not the principle of the model. The question is weather it creates a different tangency portfolio. Different investors are impacted differently by these actions and fees, thus investors are being impacted differently based on their situation.

-There exists a risk-free asset such that investors may borrow or lend unlimited amounts at the risk-free rate.



Totally not true. This fee effects the return on the tangential line. Consider how much a bank pays you in interest and how much it charges you for a loan. The impact on the model is not entirely clear but this assumption of the model is questionable.

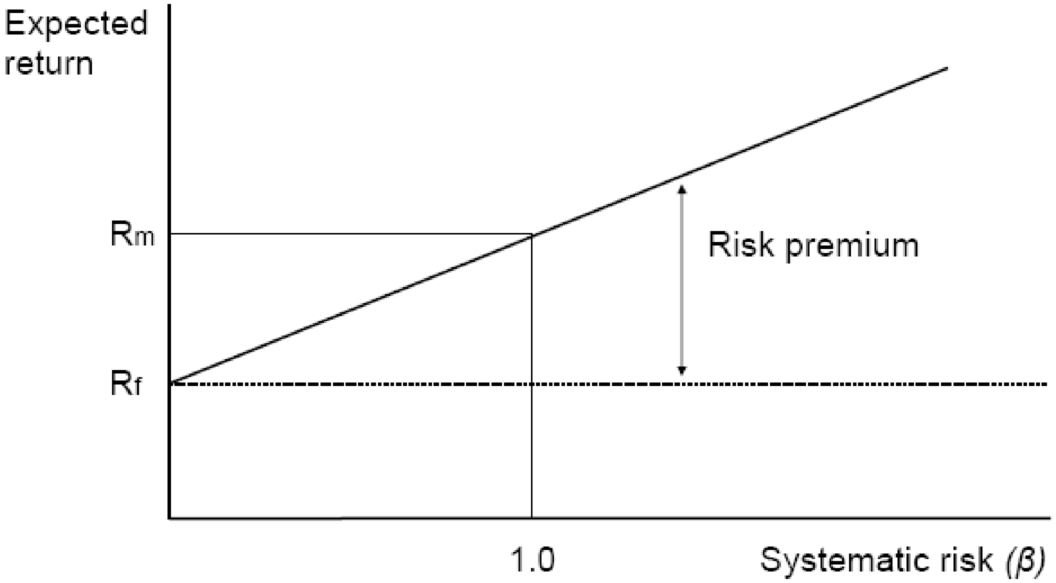
Empirical Evidence on CAPM

We know that CAPM is only an approximation. But how good an approximation is it? Does empirical evidence match theory?

- How would you test the CAPM model? Look at the implications of the theory, do they hold in real practice?

- **Testable implication 1:** Security prices lie on the security market line

Says all stocks should be on the line:



- **Testable implication 2:** Beta is the only reason expected returns differ

$$E(R_i) = R_f + \beta_i(E(R_M) - R_f)$$

β is firm specific whereas the other terms (in the CAPM eq) are the same for all firms.
Stocks with different beta must have different return. Only β is firm specific.
Test by looking at firms with the same beta and seeing if they have the same expected return.

Empirical Evidence on CAPM

1. **The SML appears to be linear.**
   This is consistent with the model.

2. **The intercept term is generally found to be higher than R_f.**
   Firms with β =0 (zero slope) have return higher than R_f, not consistent with the model, should be equal to R_f. Here we mean significantly higher.

3. **The slope of the CAPM is generally found to be less steep than posited by the theory.**
   So a firm with β =1 would have a return less than R_M. This is not consistent with the theory.

4. **There is evidence that there are other factors that affect risk premium.**
   This conflicts with the implication that β is the only thing which changes from firm to firm.

	Stock A	Stock B
	$\beta_A = 1.2$	$\beta_B = 1.2$
CAPM:	$E(R_A) = 20\%$	$E(R_B) = 20\%$
Practice:	$E(R_A) = 20\%$	$E(R_B) = 23\%$

So here we see an example where something other than β effected the return. If $E(R_B)$ is higher it must be riskier than $E(R_A)$. This is indicating that there may be other risk factors which are not included in the model. And this, of course, is not consistent with the model.

Another model attempts to bring to light these missing influences. The "**Three Factor Model**" includes the size of the firm and the market to book ratio (which allows influence of the intangible component). In this model each month all firms are ranked according to size. The average return of the **top and bottom 10%** are calculated (note that the smallest have the larger return). The difference is taken of these averages, Avg Ret Small – Avg Ret Big, the result is called **Small Minus Big (SMB)**. This single number is now a multiplier applied to all firms. The same procedure is followed wrt the **Market to Book** ratio (**HML: High Market to Book Ratio**), the higher market to book ratio firms generally have a higher return than the lower ratio firms. This number becomes another multiplier for all firms.

We now have a model that looks like this (α is the intercept):

$$R_i = \alpha + \beta_1 * RM + \beta_2 * SMB + \beta_3 * HML$$

Here we are extending CAPM ideas to other models. The three betas here represent other risk factors which make up the difference which CAPM is missing. We are talking about the size of EQUITY. β_2 and β_3 are the firms reaction to SMB and HML respectively.

Firms Value of Equity = Price of 1 share * number of shares.

So now there is a model which says we must look at 3 covariance. The percentage of "top and bottom" firms used to calculate the high and low is arbitrary, people use different values based on their beliefs. But the differences in the resulting coefficient would not be very big.

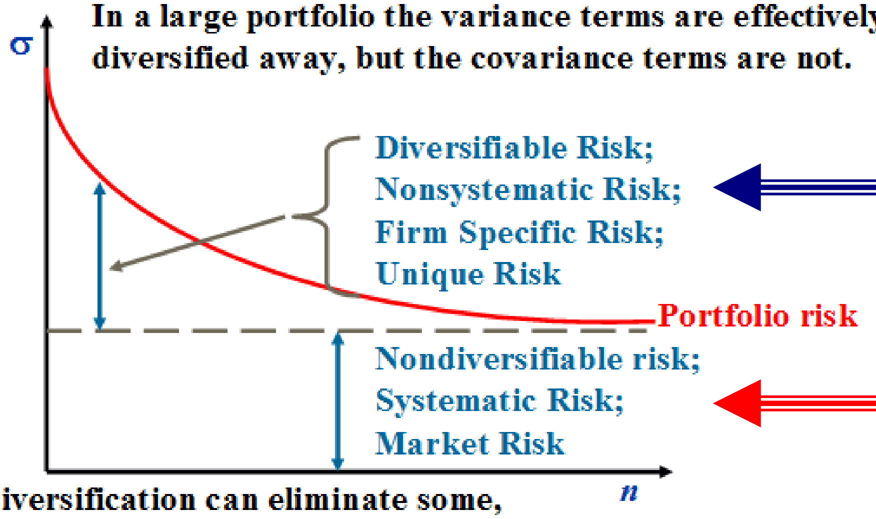
5. **New evidence suggests that there is a risk premium for unsystematic or idiosyncratic risk.**
   Systematic risk is the only risk that matters (covariance with the market). This says that the variance of a particular stock is now important, whereas according to the CAPM model only the **covariance** should be important.

Stock A		Stock B
$\beta_A = 1.2$		$\beta_B = 1.2$
$E(R_A)$	>	$E(R_B)$
$Var(R_A)$	>	$Var(R_B)$

Research finds that the expected return of stock A is higher AND the variance of stock A is higher!

In a large portfolio the variance terms are effectively diversified away, but the covariance terms are not.

This is the basis for only beta being important. But research shows variance is also important.



Can be diversified away

Only systematic matters.

Thus diversification can eliminate some, but not all of the risk of individual securities.

The problem is that not all portfolios are diversified in the same way.

Popularity of CAPM

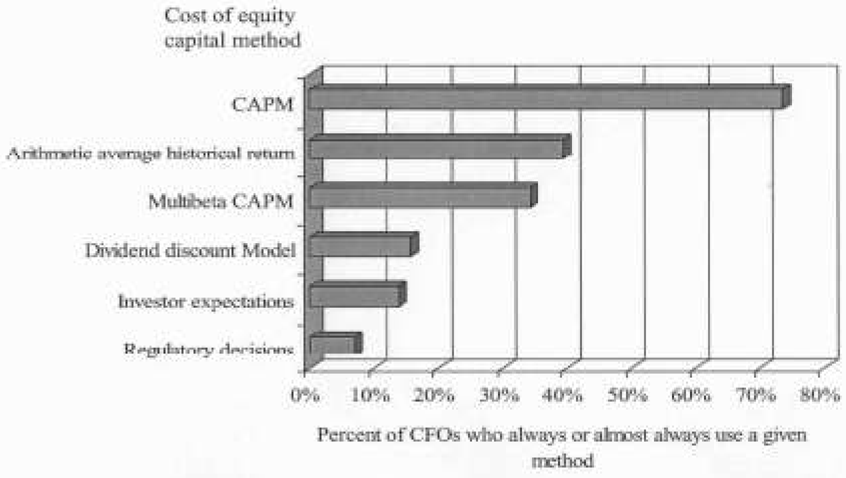


Fig. 3. Survey evidence on the popularity of different methods to calculate the cost of equity capital. We report the percentage of CFOs who always or almost always use a particular technique. CAPM represents the capital asset pricing model. The survey is based on the responses of 392 CFOs.

This chart is the result of a recent poll of CFOs. CAPM is much easier to implement in practice so it is much more popular **even if it is not the most accurate**.

Portfolios Betas

-The beta of a portfolio of assets is equal to the weighted average of the betas of the individual assets in the portfolio, where the weights are given by portfolio weights

$$\beta_P = \sum_{i=1}^{N} W_i \beta_i$$

Example

-Assume that you invest \$20 in stock A that has a beta of 0.9, \$30 in stock B with a beta of 1.2, and \$50 in stock C with a beta of 1.4 What is the beta of your portfolio? It is simply the weighted average of the individual betas.

$$\beta_P = \sum_{i=1}^{n} W_i \beta_i = \frac{20}{100} * .9 + \frac{30}{100} * 1.2 + \frac{50}{100} * 1.4 = 1.24$$

Practice questions

10.19 10.20 10.27 10.32 10.33 10.39

Return & Risk: The Capital Asset Pricing Model (CAPM)

The purpose of Chapter 10 is to more formally explain the relationship between return and risk that was first introduced in Chapter 9. This relationship is the capital asset pricing model (CAPM). Chapter 10 discusses how individuals may view a security's variance as an appropriate measure of risk. For a portfolio of assets, however, investors should view a security's contribution to the risk of the portfolio as the appropriate measure of risk. This contribution is indicated by the security's beta, which is a measure of a security's covariance risk with respect to the market portfolio. The chapter provides the necessary mathematics to calculate the expected return and variance of a portfolio containing many risky assets.

The major concepts discussed in Chapter 10 are outlined below.

- Individual securities
    - returns
    - variances
    - covariances
- Portfolios
    - returns
    - variances
    - covariances
- Efficient set for two assets
- Efficient set for many assets
    - variance
    - diversification
- Riskless borrowing and lending
    - optimal portfolio
    - capital market line
- Market equilibrium
    - homogeneous expectations
    - beta
    - characteristic line
- Capital Asset Pricing Model
    - security market line

Chapter Summary

This chapter sets forth the fundamentals of modern portfolio theory. Our basic points are these:

1. This chapter shows us how to calculate the expected return and variance for individual securities, and the covariance and correlation for pairs of securities. Given these statistics, the expected return and variance for a portfolio of two securities *A* and *B* can be written as

$$\text{Expected return on portfolio} = X_A\overline{R}_A + X_B\overline{R}_B$$
$$\text{Var(portfolio)} = X_A^2\sigma_A^2 + 2X_AX_B\sigma_{AB} + X_B^2\sigma_B^2$$

2. In our notation, *X* stands for the proportion of a security in one's portfolio. By varying *X*, one can trace out the efficient set of portfolios. We graphed the efficient set for the two-asset case as a curve, pointing out that the degree of curvature or bend in the graph reflects the diversification effect: The lower the correlation between the two securities, the greater the bend. The same general shape of the efficient set holds in a world of many assets.
3. Just as the formula for variance in the two-asset case is computed from a 2×2 matrix, the variance formula is computed from an *N*×*N* matrix in the *N*-asset case. We show that, with a large number of assets, there are many more covariance terms than variance terms in the matrix. In fact, the variance terms are effectively diversified away in a large portfolio but the covariance terms are not. Thus, a diversified portfolio can only eliminate some, but not all, of the risk of the individual securities.
4. The efficient set of risky assets can be combined with riskless borrowing and lending. In this case, a rational investor will always choose to hold the portfolio of risky securities represented by point A in Figure 10.9. Then he can either borrow or lend at the riskless rate to achieve any desired point on line *II* in the figure.
5. The contribution of a security to the risk of a large, well-diversified portfolio is proportional to the covariance of the security's return with the market's return. This contribution, when standardized, is called the beta. The beta of a security can also be interpreted as the responsiveness of a security's return to that of the market.
6. The CAPM states that

$$\overline{R} = R_F + \beta(\overline{R}_M - R_F)$$

In other words, the expected return on a security is positively (and linearly) related to the security's beta.