Risk and Risk Aversion

Do markets price in new information?

Refer to spreadsheet Risk.xls

Price of a financial asset will be the present value of future cash flows. $PV = \sum_{i=1}^{\infty} \frac{c_i}{(1+R_s)^i}$

(where c_i = are the future cash flows, R_s is the cost of capital, assuming it remains constant)

$$R_s = R_F + RPM$$

Measure risk and price risk (risk price model)

Overview of interest rates

Overview

- What do we mean by risk
- What is risk aversion

Recap of Statistics

- Suppose we have the following historical data
- We know how to calculate
  - ∗Average Return
  - ∗Variance of Returns
  - ∗Standard Deviation of Returns

Excel Functions (these use SAMPLE DATA)

AVERAGE()
VAR()
STDEV()

	SP500	MSFT
Jun	-0.91%	-2.83%
May	0.01%	2.87%
Apr	-3.09%	-5.86%
Mar	1.22%	-11.22%
Feb	1.11%	1.23%
Jan	0.05%	-4.22%
Dec	2.55%	7.67%
Nov	-0.10%	-5.53%
Oct	3.52%	8.02%
Sep	-1.77%	-0.12%
Aug	0.69%	-6.01%
Jul	-1.12%	7.20%

What is Investment Risk?

- Investment risk is related to the probability of earning a low or negative actual return. (not earning what you expected)
- The greater the chance of lower than expected or negative returns, the riskier the investment.

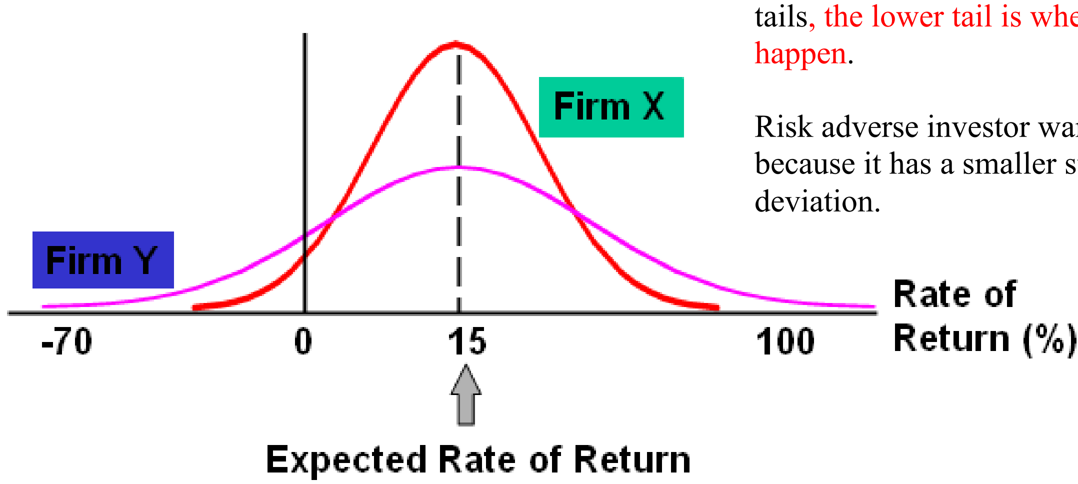
GREATER RISK = GREATER CHANCE OF NEGATIVE RETURNS

Probability Distributions

- ❑ A listing of all possible outcomes, and the probability of each occurrence.
- ❑

Two firms, Y has more risk, this is seen by it's larger tails, more area in it's tails, the lower tail is where the losses happen.

Risk adverse investor wants firm X because it has a smaller standard deviation.



- ❑ Can be shown graphically.

These curves show all probable outcomes and they are pretty close to normal. Good working model. Tails are a little fat but we'll have to deal with it.

Given a choice of X and Y above, the only way you would prefer Y is if you are a risk lover. Both give the same expected return but firm X gives us that return with less standard deviation.

Standard Deviation is an important measure of risk but it is not final answer.

Given a choice between one investment or another, standard deviation is the appropriate measure of risk. If I can be diversified, then the important factor in the risk of a particular investment is BETA and not standard deviation.

However, even if I hold a well diversified portfolio, risk is still the standard deviation of that portfolio's return. Want to have an appreciation for when standard deviation is important and when beta is important.

Firm Y has the greater standard deviation, the flatter the distribution the greater the standard dev.

What do we mean by Risk Adverse?

There are many ways of defining risk aversion.

EXAMPLE:

Which do we prefer?
1) we get $10,000 each.
2) flip coin, heads get $20,000, tails get nothing.

Most people will want choice 1.
Now increase 10 1) = $10,000,000 and 2) = $200,000,000.
Most probably still want 1).

A risk adverse investor prefers the risk adverse payout with the same expected value.

EXAMPLE:

1) certain $1.5 million. 2) $0, how many want? $1 mill, how many want? $2.5 mill, how many want? Keeps going up. Expected payout of the gamble is increasing. If at the point where choice 2 becomes $5 million I prefer choice 2, then at that point I am making an assessment. What I am deciding is that the expected value of choice 2 is equal to $2.5 million. On average, if we played this game many times, that would be the average payout or choice 2 given these odds.

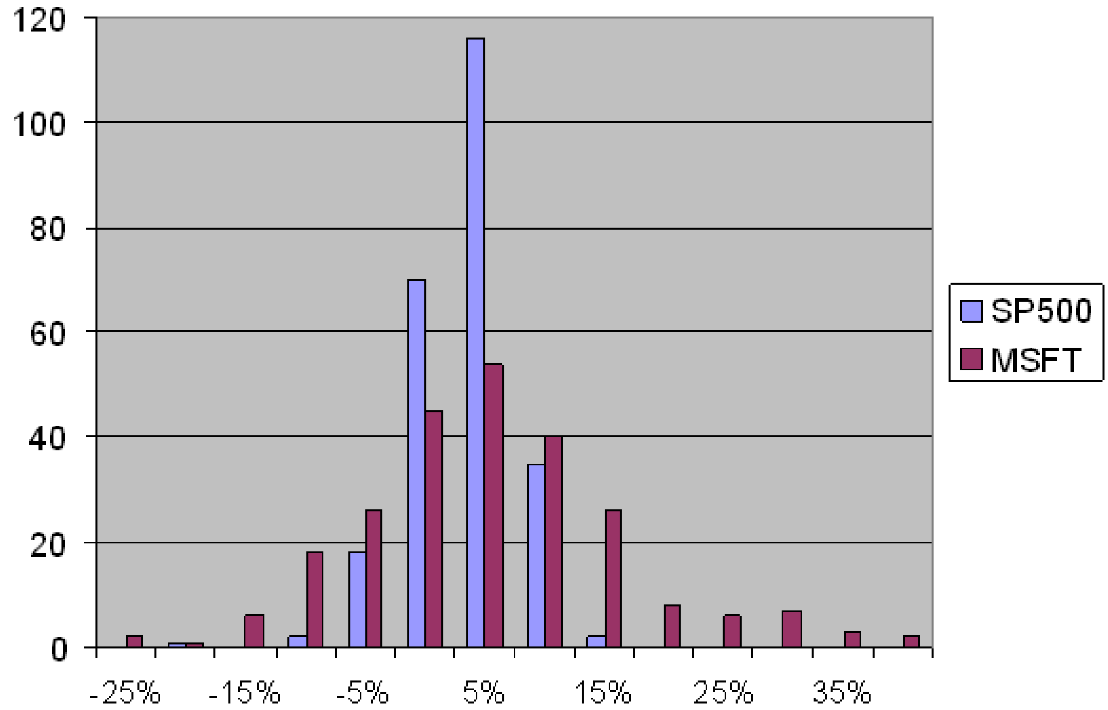
E(2) = $2.5 MILLION (over many trials)

What we have found is that the…

RISK PREMIUM = $2.5 million - $1 million = $1.5 million

If $5 million is where I jumped then I need a $1.5 million risk premium in order to take the chance. (on average). This is the additional payout I need in order to take the gamble.

Probability Distributions

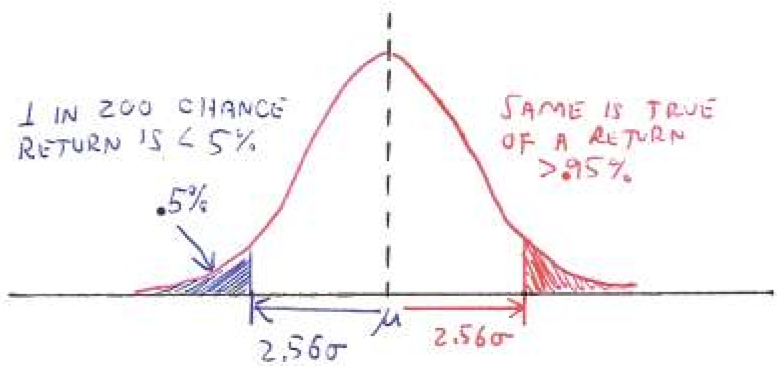


This graph shows the monthly return on Microsoft and S&P500 for 20 years. This is a histogram. S&P500 made 5% return about 20 times out of the data examined. Seems to be peaked, less risk (?). Average return of MS seems to be a bit higher. Appears that S&P has lower standard deviation but MS has higher return.

Now which is the better investment? We don't know how to price that additional risk.

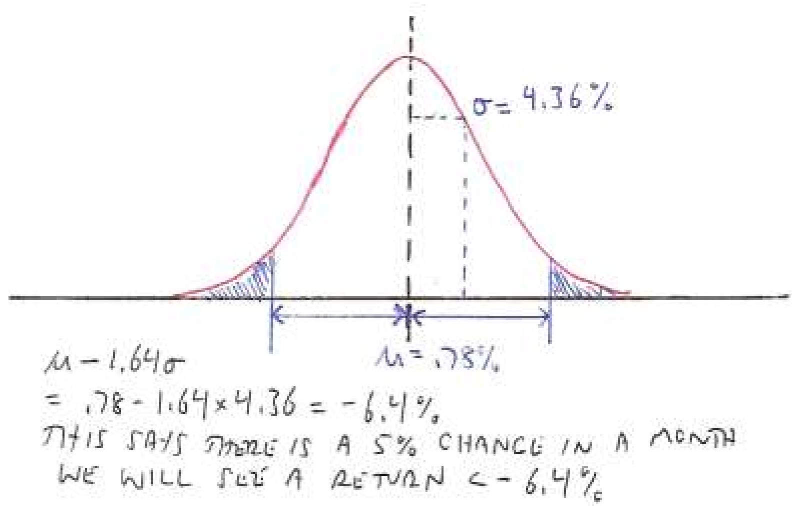
Normal Distribution

- For a normal distribution, we have the following probabilities
  - Probability actual event is 2.56 standard deviations above or below the mean is 0.5% in each case.
  - Probability actual event is 2.33 standard deviations above or below the mean is 1.0% in each case.
  - Probability actual event is **1.64** standard deviations above or below the mean is **5.0%** in each case.
  - Probability actual event is 1.28 standard deviations above or below the mean is 10% in each case.



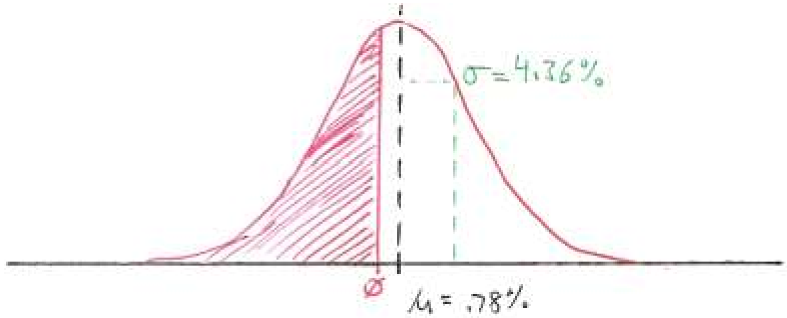
Normal Distribution – S & P 500

- If monthly returns of the S & P 500 are normally distributed with
  - mean = 0.78% and, σ = 4.36%
- Then there is a 5% chance (1.64σ) that the actual return in a given month will be less than –6.4%
- Another way to put this is that there is only a 95% chance that actual return in a given year will be greater than –6.4%



❑ Turning the question around

- What is the probability of suffering a monthly loss
- Use the Excel function NORMDIST with the following entries
  - x = 0%, mean = 0.78%, standard_dev = 4.36%, and cumulative = True
  - Answer = 42.9%
- In other words there is a 57.1% chance of seeing a monthly profit



What is the probability of a loss in a given month? Area up to the mean is ½, use normdist() to find area below 0.0%. PUT IN PERCENT SIGNS, ALWAYS USE CUMULATIVE=TRUE

NORMDIST(0%, .78%, 4.36%, TRUE) = 42.9% chance in any particular month of being a loss month.

❑ Using our historical sample we will see if these answers seem right

- Using the function COUNTIF, with the first entry the cells containing the data, and the second entry "<-6.4%" (The quotation marks are required), I find 14 occurrences of losses greater than -6.4%
- With a second entry of "<0" I find 91 occurrences of losses
- There are 244 data points
- Based on history there is a 14/244 = 5.7% chance of a loss greater than -6.4%, and
- There is a 91/244 = 37.3% chance of a loss (Results are pretty close)

This method does not rely on the data being normally distributed.

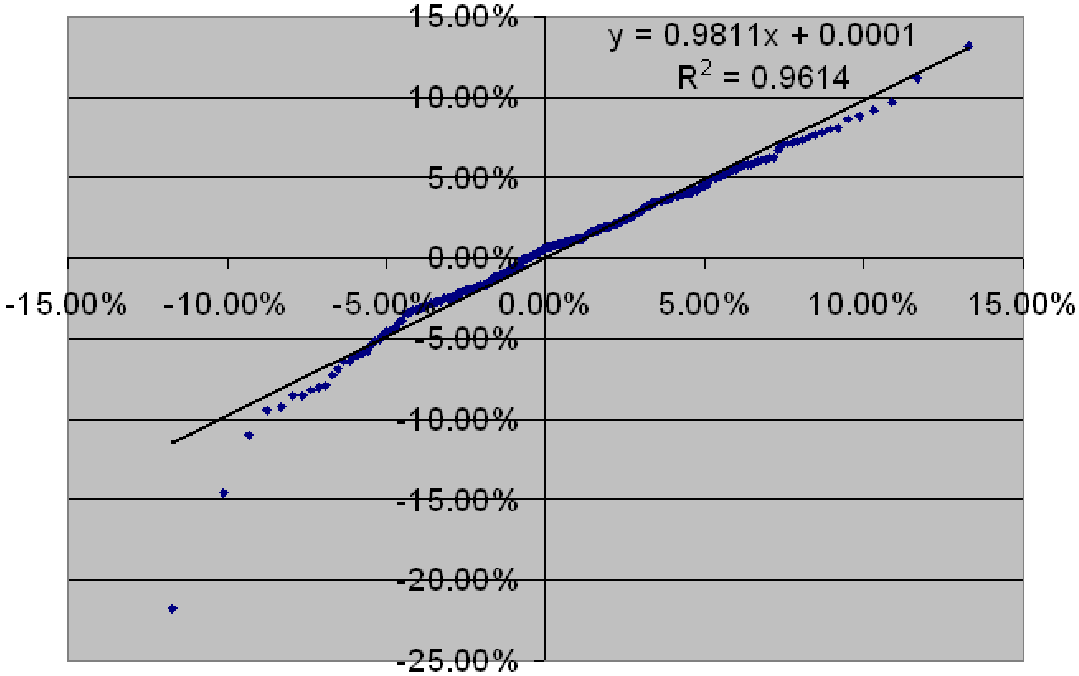
In this method we use the data directly to calculate the probability of a loss in a month. Count the number of outcomes less than or equal to -6.4% and divide by the total number

COUNTIF(C2:C245, "-6.4%") returns the result of 14 events out of 244.

This procedure called **Value At Risk Analysis**.

Test for Normality

- ❑ The easiest way is to generate 244 data points form a Normal distribution with the same mean and standard deviation
  - ∗ Use the function NORMINV with mean of 0.78% and standard deviation of 4.36%
  - ∗ Probabilities are 0.5/244, 1.5/244, 2.5/244,… , 243.5/244
- ❑ Sorting the original data the following plot is obtained



Why did the secondary method of calculating the probability not match the normal distribution method? Could be that the data is not normal. Test the data.

The closer to a straight line the more normal the data. The deviation on the left end is typical of most asset returns. This, the left end, is the area of bad losses so it's not good that our data departs from normal here. This means that when we do loss we will loss more than predicted. It's a big problem that we cannot rely on the normal model in this loss region.

An advantage may be to use actual historical data when available.

Selected Realized Returns, 1926 – 2001

	Average Return	Standard Deviation
Small-company stocks	17.3%	33.2%
Large-company stocks	12.7	20.2
L-T corporate bonds	6.1	8.6
L-T government bonds	5.7	9.4
U.S. Treasury bills	3.9	3.2

Standard deviations are moving high to low.

Source: Based on *Stocks, Bonds, Bills, and Inflation: (Valuation Edition) 2002 Yearbook* (Chicago: Ibbotson Associates, 2002), 28.

L-T: long term, treasury bills are short term. Here we are seeing annual returns for 76 years. This is showing us that investors are compensated for risk.

Stocks carry more risk than bonds. Small company stocks considered riskier. Corporations are riskier than government bonds.

Long Term Government Bonds carry Reinvestment Risk. Hanging on to these things a long time, rates may change.

High Risk to Low Risk seems to coincide with high returns to low returns and also seems to coincide with high standard deviation and low standard deviation. Seems to make sencse based on history.

Covariance for Historical Data

- Now we will calculate covariance
- For a sample of paired data, the following statistic measures the covariance of the two variables:

$$Covariance = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)$$ (SAMPLE DATA)

Looking at two variables, how closely linked? Any co-variability? Do they move in the same way? Could be comparing two stocks for example.

Covariance for Historical Data (how to calculate)

	SP500	MSFT	(xi-xbar) for SP500	(yi-ybar) for MS	(xi-xbar)(yi-ybar)
Jun	-0.91%	-2.83%	-1.09%	-2.10%	0.02%
May	0.01%	2.87%	-0.17%	3.60%	-0.01%
Apr	-3.09%	-5.86%	-3.27%	-5.13%	0.17%
Mar	1.22%	-11.22%	1.04%	-10.48%	-0.11%
Feb	1.11%	1.23%	0.93%	1.97%	0.02%
Jan	0.05%	-4.22%	-0.13%	-3.49%	0.00%
Dec	2.55%	7.67%	2.37%	8.40%	0.20%
Nov	-0.10%	-5.53%	-0.27%	-4.80%	0.01%
Oct	3.52%	8.02%	3.34%	8.75%	0.29%
Sep	-1.77%	-0.12%	-1.95%	0.62%	-0.01%
Aug	0.69%	-6.01%	0.52%	-5.28%	-0.03%
Jul	-1.12%	7.20%	-1.30%	7.93%	-0.10%
Avg:	0.18%	-0.73%			
				SUM --->	0.0046

Correlation: 0.36729

Div by n-1 gives covariance = **0.0004**

Correlation

- For a sample of paired data, the following statistic measures the correlation of the two variables:

$$Correlation = \frac{Covariance\ of\ X\ and\ Y}{Standard\ Deviation\ of\ X * Standard\ Deviation\ of\ Y}$$

Correlated if there is a pattern between the two. The scale is difficult to interpret so we divide by the standard deviation of each parameter.

Correlation Between Two Securities

Positively correlated: $0 < \rho < 1$

Perfectly Positively correlated: $\rho = 1$

Negatively correlated: $-1 < \rho < 0$

Perfectly Negatively correlated: $\rho = -1$

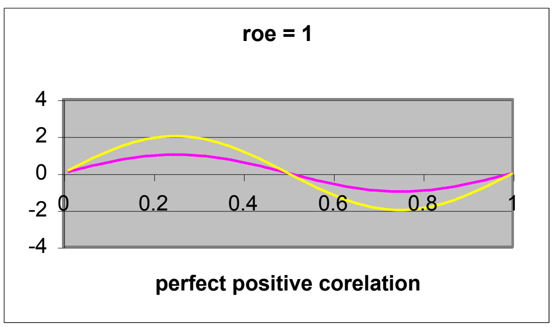
Uncorrelated: $\rho = 0$

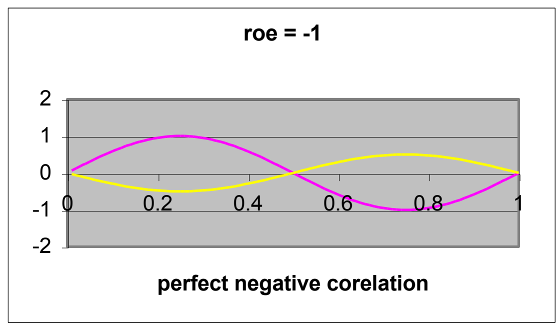
Correlation Between MSFT and S & P 500

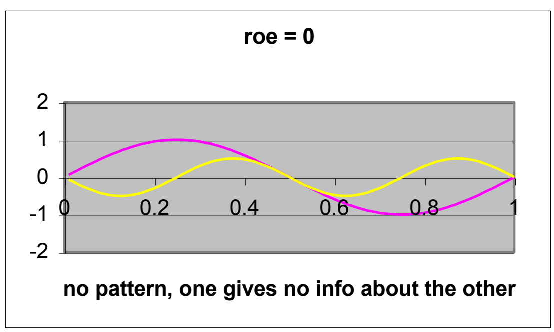
- Using the function CORREL we have

  $\rho = 0.549$

  CORELL(Range,Range), function uses the raw data!







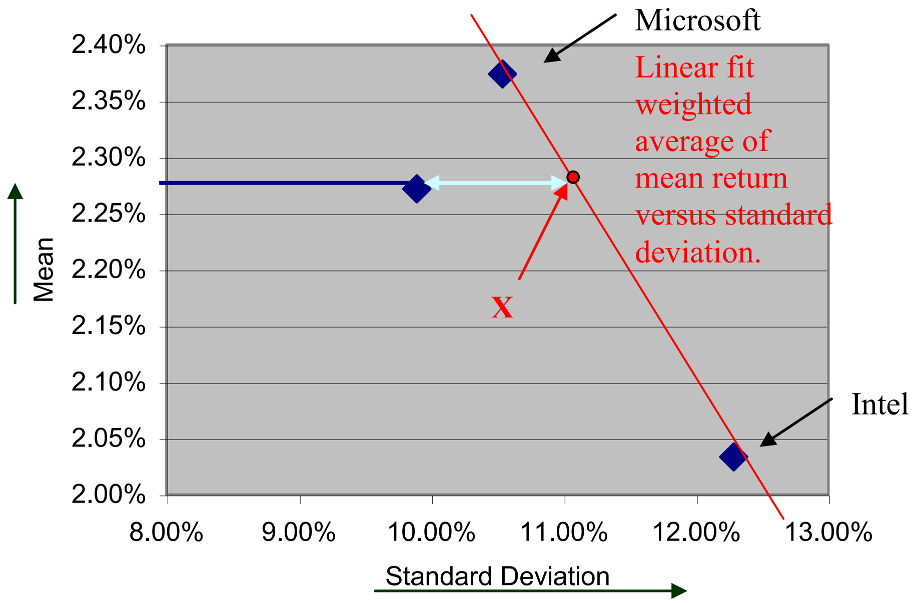
Perfect correlation only means that the two variables are moving in the same direction. It makes no inference about the magnitude of the movement, only direction.

Portfolios

- Suppose many years ago I had formed a portfolio containing two stocks, Intel and Microsoft
  - Suppose I had invested 30% of my wealth in Intel and 70% in Microsoft
  - This would lead to the following

	Intel	Microsoft	Portfolio
Mean	2.03%	2.37%	2.27%
Std Dev	12.29%	10.54%	9.89%
CoV	6.04	4.44	4.35

Sheet 3 of spreadsheet, 20 years of monthly data for Intel and MS. The columns are matched pairs. Is there any benefit to including two stocks in a portfolio? We will calculate 30% of Intel and 70% of MS into new columns, this weights the terms by the amount each is represented in the portfolio. $COV = \frac{\sigma}{\mu}$



Plotting $(\sigma,\ \mu)$, weighted standard deviation and mean, of Intel and Microsoft.
Then X represents the point with
mean return = $30\%\ \mu_{Intel} + 70\%\ \mu_{Microsoft}$ and
Standard deviation = $30\%\ \sigma_{Intel} + 70\%\ \sigma_{Microsoft}$.
Less standard deviation (30% in Intel case) means less risk. But we still get the weighted average of the mean. (because $\rho < 1$).

Putting the stocks together gives us a level of return (mean return) with a smaller standard deviation (less risk). The ρ value dampens the risk swings.

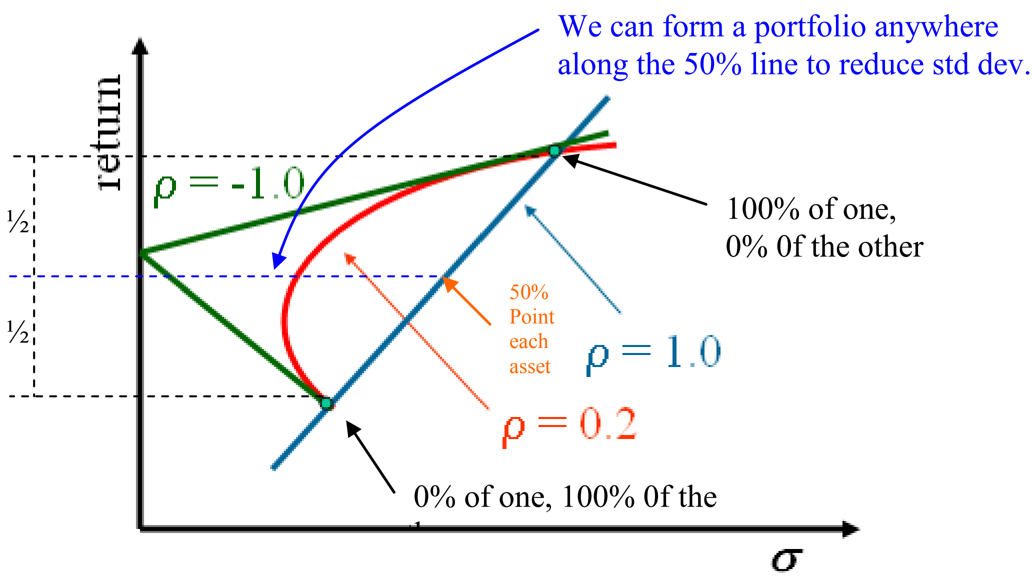
mean return = $30\%\ \mu_{Intel} + 70\%\ \mu_{Microsoft}$ and standard deviation = $30\%\ \sigma_{Intel} + 70\%\ \sigma_{Microsoft}$ specifies a new distribution which describes the two stocks in the proportions which they are represented in the portfolio, describes the risk of the portfolio.

The portfolio, as a new asset, carries over the value of the means exactly, it's just the weighted average of the means. Because of the lack of correlation between the assets (one stock may be up when another is down) it erases some of the swings.

If roe were +1 we would get nothing (none of the swing erasure), we would only get the weighted average of the standard deviation. We would eliminate no risk. But because some of the ups and downs have been damped down (because the correlation is NOT +1) we move away from the X value on the line toward the point to the left. In this way we loss some of the standard deviation (risk) which is a good thing.

When you look at how valuable an asset is what you really want to understand is what it does in terms of risk elimination. An asset with a very low correlation, it is a very valuable asset, we can use it to reduce risk. If roe were negative you would move even farther away. If roe were -1 you could actually put the portfolio at the standard deviation = 0 point. You can create a portfolio where standard deviation actually becomes zero. As long as it is not minus 1 you never entirely eliminate risk. And even if roe is -1 you'd have to pick your portfolio carefully.

Two-Security Portfolios with Various Correlations



If correlation (roe) is +1 the portfolios are on a **straight line** joining the two points, this is because in this case **we just have the weighted average of mean and standard deviation**. At roe = +1 there is no benefit to diversification, at +1 they are moving together in the same direction, there is not variability eliminated.

The only elimination of risk we get is by mixing the two assets together and moving from one standard deviation to another. We can form a portfolio anywhere along the 50% line (blue dashed). The nice thing is we get the same return but get to eliminate some risk (reduce standard deviation).

The green triangle type lines represent an extreme of perfect negative correlation at which point we are at zero standard deviation, no risk all return. The ups of one stock would completely negate the downs of the other. But this does not exist in the real world.

Can't quite think of the lowest std dev point as being the best, have to consider the trade off (coming up).

Efficient Sets And Diversification

- The expected return on a portfolio is the weighted average of the expected returns on the individual securities
- As long as □ < 1, the standard deviation of a portfolio of two securities is less than the weighted average of the standard deviations of the individual securities

So in the real world we will always have: $\sigma_\rho < w_1\sigma_1 + w_1\sigma_1 + \cdots$

If we have assets 1 through n in a portfolio the weights w_1 through w_n will sum to 1, $w_1 + w_2 + \cdots \quad 1$ and we have mean returns $\mu_1,\ \mu_2,\ \cdots$ then

$\mu_\rho < w_1\mu_1 + w_1\mu_1 + \cdots$

$\sigma_\rho < w_1\sigma_1 + w_1\sigma_1 + \cdots$

Keep in mind that the weight values, w, do NOT have to be positive. Barrowing and selling short are examples that would result in negative w's.

Identity:

$\sigma_\rho = w_1\sigma_1 + w_1\sigma_1 + \cdots$ if and only if $\rho_{1,2} = \rho_{1,3} = \rho_{1,4} \cdots \quad +1$

Variance or Covariance?

- Suppose we have N securities each with the same variance and the same covariance with the others. (use the averages instead)
- Suppose we form an **equally weighted** portfolio of these securities. (equal % in each)

  Therefore $w_1 = w_2 = \ldots = w_N$
- The variance of the portfolio will be given by…

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + \ldots + w_S^2\sigma_S^2 + 2w_1w_2\,\sigma_{12} + 2w_1w_3\,\sigma_{13} + \ldots + 2w_1w_S\,\sigma_{1S} + 2w_2w_3\,\sigma_{23} + 2w_2w_4\,\sigma_{24} + \ldots + 2w_2w_S\,\sigma_{2S} + \ldots + 2w_{S-1}w_S\,\sigma_{S-1S}$$

- There are N equal variance terms , call them var, and $N^2 - N$ covariance terms, call them covar. Assuming all $\sigma_{j,p}$ are equal and all weight terms w_i and $w_{i,k}$ are equal (this is saying there are N assets each with weight value $\frac{1}{N}$). Then …

$$\sigma_p^2 = N(1/N)^2\text{var} + (N^2 - N)(1/N)^2\text{covar}$$

which reduces to …

$$\sigma_p^2 = \text{var}/N + (1 - 1/N)\text{covar}$$

as N becomes large…

$$\sigma_p^2 = \cancel{\text{var}/N} + (1 \cancel{- 1/N})\text{cover}$$

leaving only …

$$\boxed{\sigma_p^2 = \text{covar}}$$

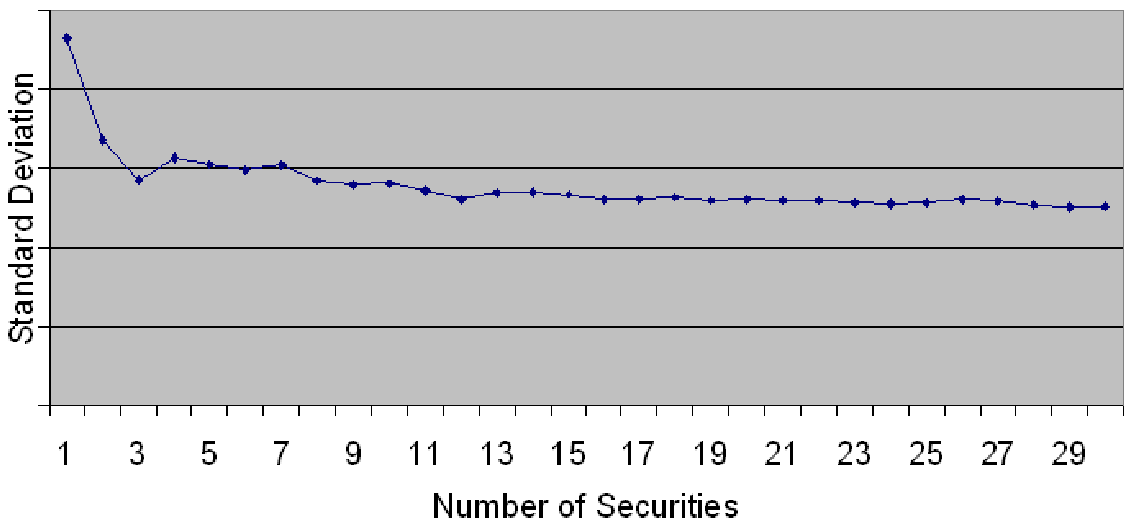
- So as **N gets bigger**, the individual variance of a stock becomes less and less important and **the covariance term dominates**.

As we add stocks to the portfolio the individual σ's become irrelevant.
[tape 2 index 2]

Variance or Covariance

- ❑ The below chart was constructed in the following way.
  - ∗ Select the Dow 30 monthly returns for last 15 years
  - ∗ Form portfolios containing the one security, then the two, and so on.
- ❑ On average we have standard deviation of the individual security is 8.4%.
- ❑ On average we have the $\sqrt{\sigma_{x,y}}$ (covariance$^{0.5)}$ of the individual security is 4.9%.

Keep increasing the number of stocks in each portfolio (which is a point on the graph) until you reach the last data point which is a portfolio with 1000 stocks.



[tape 2 index 3]

As we do this we find that the covariance becomes important, not the variance. This is the reason why diversification works. The points trend to an average value of 4.9%. But we can see we get most of the benefit at 5 stocks! At 20 stocks there is no further gain to adding to the portfolio, no more benefit of diversification.

EXAM

Return Statistics Ex-ante **(look to future)**

Expected Return of a Security:

$E(R) = p_1R_1 + p_2R_2 + \ldots + p_NR_N$

(elements N represent different states of the world in the future)

Variance of a Single Security:

$\sigma^2 = p_1[R_1 - E(R)]^2 + p_2[R_2 - E(R)]^2 + \ldots + p_N[R_N - E(R)]^2$

Standard deviation of a Single Security: σ

Covariance Between Two Securities:

$\sigma_{AB} = p_1[R_{A1} - E(R_A)][R_{B1} - E(R_B)] + \ldots + p_1[R_{AN} - E(R_A)][R_{BN} - E(R_B)]$

Correlation Between Two Securities: $\rho_{AB} = \dfrac{\sigma_{AB}}{\sigma_A \sigma_B}$

Here we are looking at the probability of a future economic state times the return we expect if that state come true.

This is not that same as FORCAST DATA which we examine below.

Example of Return Statistics

Outcomes	Probability (p)	R_A	R_B
Boom	0.25	20%	5%
Normal	0.5	10%	10%
Bust	0.25	0%	15%

N

Use this table of future states of the economy and the probability that we believe the particular state will arise. RA and RB are the returns we expect corresponding to a particular security.

These equations are of the form listed at the top of the page. Here we are taking the 3 R terms fos a particular future state and averaging them for the expected value of return of that future state, E(R).

$E\{r_A\} = (.25)(.20) + (.5)(.10) + (.25)(.00) = .10$

$E\{r_B\} = (.25)(.05) + (.5)(.10) + (.25)(.15) = .10$

$\sigma_A^2 = (.25)(.20 - .10)^2 + (.5)(.10 - .10)^2 + (.25)(.00 - .10)^2 = .00500$

$\sigma_B^2 = (.25)(.05 - .10)^2 + (.5)(.10 - .10)^2 + (.25)(.15 - .10)^2 = .00125$

(these sigma's are the weighted variances)

$\sigma_A = (.00500)^{1/2} = .07071 = 7.071\%$

$\sigma_B = (.00125)^{1/2} = .03536 = 3.536\%$

$\sigma_{AB} = (.25)[(.20 - .10)(.05 - .10)] + (.5)[(.10 - .10)(.10 - .10)] + (.25)[(.00 - .10)(.15 - .10)] = -0.0025$

$\rho_{AB} = -0.0025 / (0.07071)(0.03536) = -1$ ←PERFECT NEGATIVE CORR.

Diversification

- Suppose in the previous example we invest $100 in security A and $200 in B. Dollar returns under each possible outcome are:

Outcome	Probability	$100 in A	$200 in B	$300 in A&B	%Return on
Boom	0.25	$120	$210	$330	10%
Normal	0.5	$110	$220	$330	10%
Bust	0.25	$100	$230	$330	10%

In this example all the investment options are returning 10%. We invest 1/3 of our money in A and 2/3's of our money in B. In this portfolio the return is fixed regardless of the state of the world. But this is only because roe=-1, not realistic. But since roe=-1 we are forcasting a portfolio without risk!

Expected return of the portfolio is 10%

From previous page:

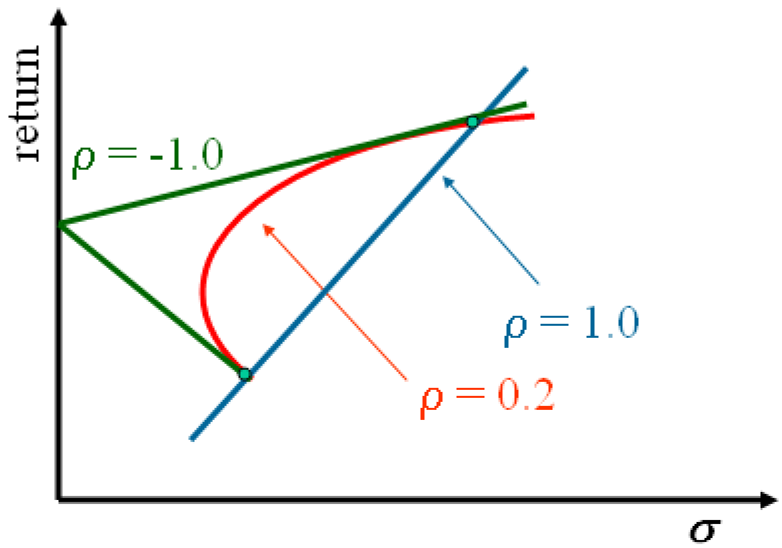
$E\{r_A\}= (.25)(.20)+(.5)(.10)+(.25)(.00) = .10$, so 100*1.1=120

$E\{r_B\}= (.25)(.05)+(.5)(.10)+(.25)(.15) = .10$, so 200*1.1=210

The variance of the portfolio is 0%

The standard deviation of the portfolio is 0%

Two-Security Portfolios with Various Correlations



Return and Risk for Portfolios

- Expected Return of a Portfolio:
  $E(R_p) = w_1E(R_1) + w_2E(R_2) + \ldots + w_SE(R_S)$

- Variance of a Portfolio:
  $\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + \ldots + w_S^2\sigma_S^2 +$
  $2w_1w_2\,\sigma_{1,2} + 2w_1w_3\,\sigma_{1,3} + \ldots + 2w_1w_S\,\sigma_{1,S} +$
  $2w_2w_3\,\sigma_{2,3} + 2w_2w_4\,\sigma_{2,4} + \ldots + 2w_2w_S\,\sigma_{2,S} + \ldots$
  $\ldots + 2w_{S-1}w_S\,\sigma_{S-1,S}$

Same as we've done above but here we are dealing with FORECAST DATA, **E(R)** as opposed to returns, r, times a probability of that return.

Two Asset Portfolio

- **Expected Return** of a Two Asset Portfolio:
  $E(R_p) = w_AE(R_A) + w_BE(R_B)$
- **Variance** of a Two Asset Portfolio:
  $\sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_Aw_B\,\sigma_{AB}$

Example

In previous example we invested \$100 in stock A and \$200 in stock B. So (this stocks investment/total investment)

(the weight values are determined by the amount (percentage) of investment in that stock)

$$w_A = \frac{100}{300} \quad \text{and} \quad w_B = \frac{200}{300}$$

(each had a 10% return)

Therefore…

$$E(R_P) = (1/3)E(R_A) + (2/3)E(R_B)$$
$$= (1/3)(.10) + (2/3)(.10) = 0.10$$
$$\sigma_p^{\,2} = w_A^{\,2}\sigma_A^{\,2} + w_B^{\,2}\sigma_B^{\,2} + 2\, w_A\, w_B\, \sigma_{AB}$$
$$= (1/3)^2(.00500) + (2/3)^2(.00125) + (2)(1/3)(2/3)(-.0025) = 0$$

Negative covariance

Variance of 0 means standard deviation is 0 which means this is an example of perfectly negative correlation, $\rho = -1$. (no risk, not realistic)

Investor Attitude Towards Risk

- **Risk aversion** – assumes investors dislike risk and require higher rates of return to encourage them to hold riskier securities.
- **Risk premium** – the difference between the return on a risky asset and less risky asset, which serves as compensation for investors to hold riskier securities. The additional return I need in order to accept the risk and make the investment.
- **Risk Neutral** – Indifferent to risk.

A Quick Illustration


Choice 1: throw a fair die, if 6 we get $6 million otherwise we get 0.
Choice 2: given $1 million.

We take choice 2 because we are risk adverse and this is a substantial amount of money. If choice 2 is taken away we are still willing to play, still happy but not as happy. We stand to loss nothing but do not have the option of the sure $1 million.

How can we reduce risk? One way is to come together as a group and each roll, splitting our winning evenly among ourselves. As a group of 6 we have the expectation that in 6 rolls at least 1 person will win, and we will split the winning for $1 million each.

Modify the game. We will each still roll the die but before we do a coin will be flipped. If the coin is heads we go forward with the game. If the coin is tails we lose, no flipping, get nothing. Is there still value in playing as a group? Yes, because if heads and we play the game we have eliminated risk by playing as a group.

The risk associated with flipping the coin is called **SYSTEMATIC RISK** because it effects everything in the game. The risk associated with the die roll is called **UNSYSTEMATIC RISK**. In financial markets it is called **Company or Asset Specific Risk**.
[tape 2 index 4]

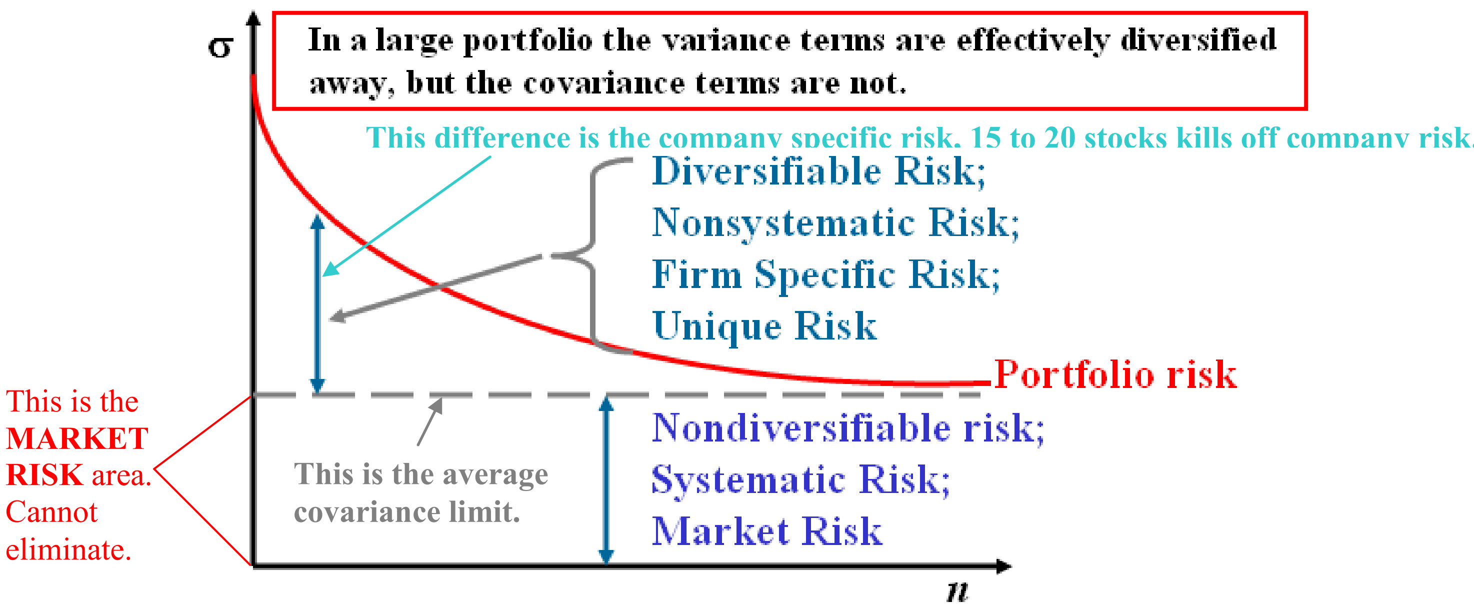
Systematic versus Unsystematic Risks

- What changes stock prices?
- How can we categorize these things?
- **Total risk of individual security = portfolio (systematic) risk + unsystematic (diversifiable) risk**

News: is it market moving news or company specific news?

News, new information, effects stock prices but not entirely, only about 30% of movement from news.

Portfolio Risk as a Function of the Number of Stocks in the Portfolio



Thus diversification can eliminate some, but not all of the risk of individual securities.

Adding stocks to a portfolio in a random manner. We start with a high standard deviation but as we add stocks std dev declines until it reaches the MARKET RISK limit.

σ = Total Risk = Company Specific Risk + Market Risk

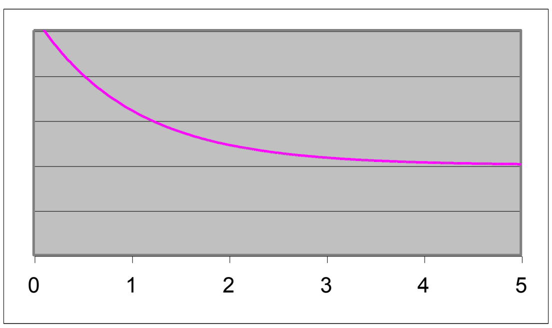
(disappears as we add more stocks) this is β

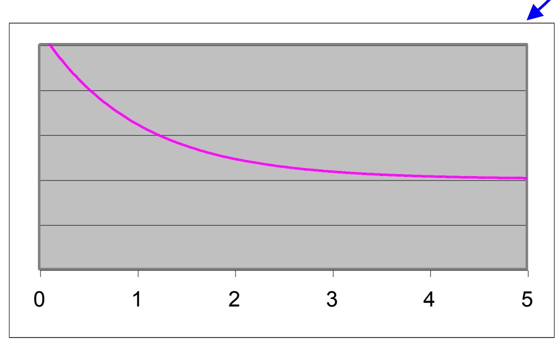
EXAM

Creating a Portfolio

- Beginning with one stock and adding randomly selected stocks to portfolio
- σ_p decreases as stocks added, because they would not be perfectly correlated with the existing portfolio.
- Expected return of the portfolio would remain relatively constant.
- Eventually the diversification benefits of adding more stocks dissipates (after about 10 stocks), and for large stock portfolios, σ_p tends to converge to ≈ 20%.

Randomly form a portfolio by randomly adding stocks. Now repeat this process and average out over many portfolios. You will have outliers but the more stocks you add and average the closer you will be to landing on these plots, the smoother the graph will be. Risk always starts high and comes down as we add diversify.





Breaking Down Sources of Risk

Stand-alone risk = Market risk + Firm-specific risk

(aka: Total Risk)

- ***Market risk*** – portion of a security's stand-alone risk that cannot be eliminated through diversification. Measured by beta. Residual σ becomes β.

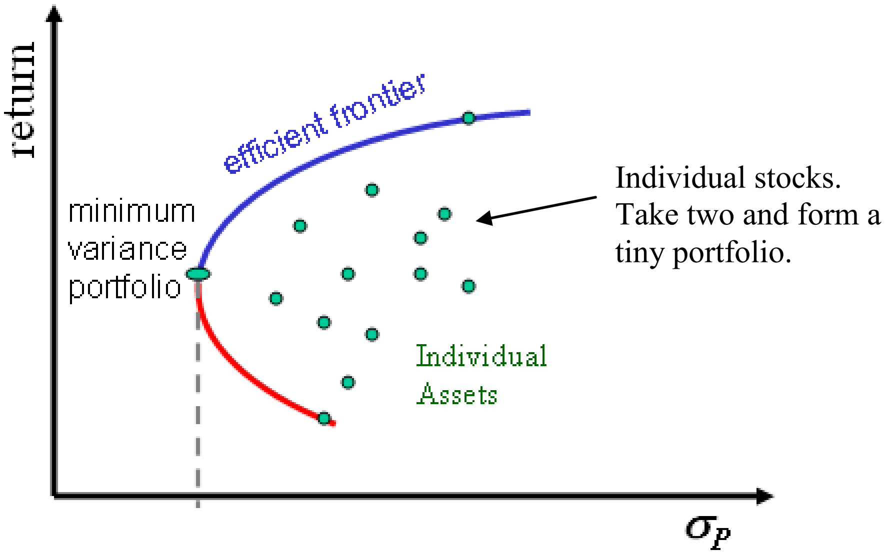
- ***Firm-specific risk*** – portion of a security's stand-alone risk that can be eliminated through proper diversification.

Failure to Diversify

- If an investor chooses to hold a one-stock portfolio (exposed to more risk than a diversified investor), would the investor be compensated for the risk they bear?
  - NO! Not rewarded for not removing risk.
  - Stand-alone risk is not important to a well-diversified investor.
  - Rational, risk-averse investors are concerned with σ_p, which is based upon market risk.
  - There can be only one price (the market return) for a given security.
  - No compensation should be earned for holding unnecessary, diversifiable risk.

An investor not diversified is not compensated for keeping risk.

The Efficient Set for Many Securities



This is the set of portfolios we can form by taking risky investments. Expanding this process eventually leads to the efficient frontier.

A rational investor will never hold a portfolio that is not on the **EFFICIENT FRONTIER** and it has to be above the minimum variance portfolio, in the blue part.

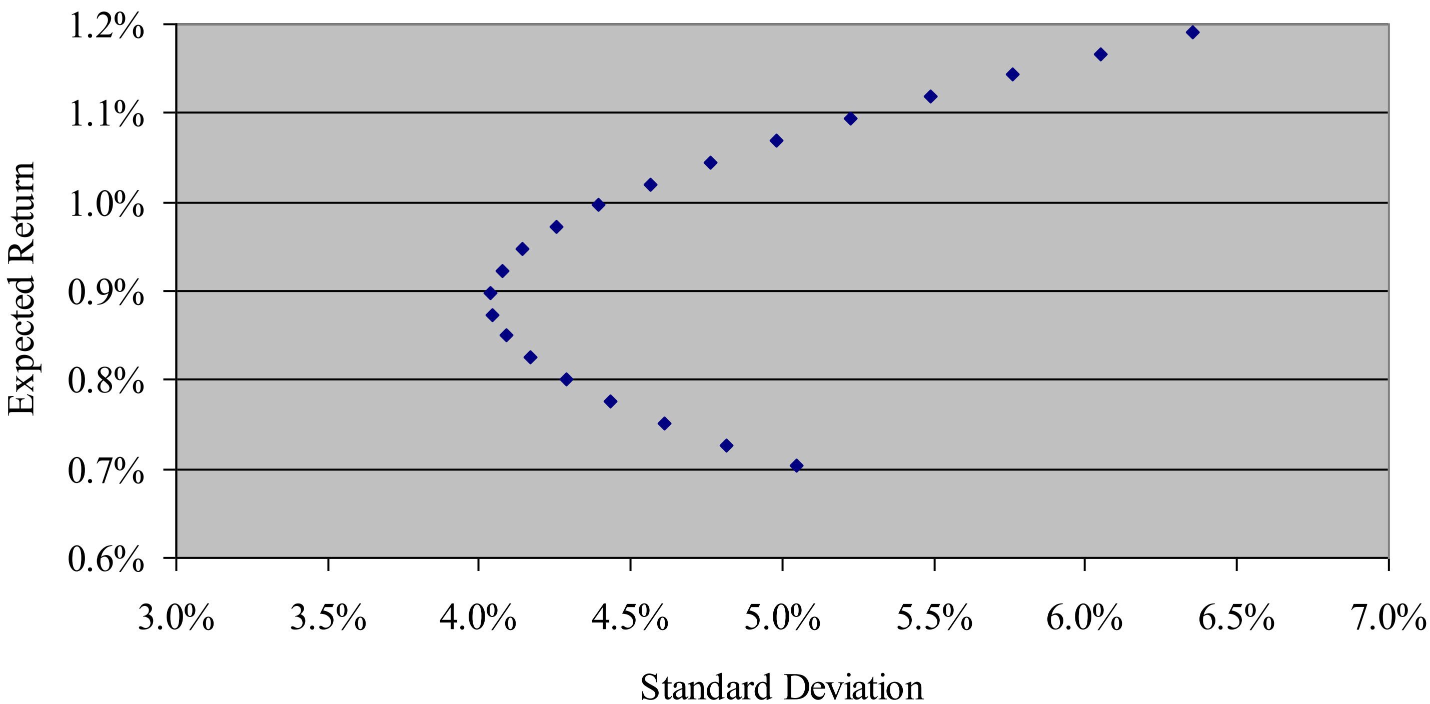
Foreign Investment Example

- I have taken monthly returns for the SP500 and a Swiss market index for the last 12 years.
- I formed portfolios with varying weights in each asset.
- The table shows the results of this analysis.

	Swiss	SP500
Mean	1.1%	0.8%
Var	0.0025	0.0017
Std Dev	5.0%	4.2%
Covar		0.0013

Weight	E{r}	Std Dev
-50%	1.19%	6.35%
-40%	1.17%	6.05%
-30%	1.14%	5.76%
-20%	1.12%	5.49%
-10%	1.09%	5.23%
0%	1.07%	4.98%
10%	1.05%	4.76%
20%	1.02%	4.56%
30%	1.00%	4.39%
40%	0.97%	4.25%

Weight	E{r}	Std Dev
50%	0.95%	4.14%
60%	0.92%	4.07%
70%	0.90%	4.04%
80%	0.87%	4.04%
90%	0.85%	4.09%
100%	0.83%	4.17%
110%	0.80%	4.29%
120%	0.78%	4.43%
130%	0.75%	4.61%
140%	0.73%	4.82%



Riskless Borrowing and Lending

- Suppose we form portfolios of a risky asset (say a stock) and a riskless asset (say a government bond).
- Suppose we invest with weights w and 1-w (where w is the weight of the stock).
- The expected return of the portfolio is:

$$E\{r_P\} = w \times E\{r_S\} + (1 - w) \times r_f$$
$$= r_f + w \times [E\{r_S\} - r_f]$$

- The standard deviation of the portfolio is:

$$\sigma = \sqrt{w^2\sigma_S + (1-w)^2 \sigma_f + 2w(1-w)\sigma_{Sf}}$$

Introduce a riskless asset, it guarantees a return over it's investment horizon. Examples would include T-Bills or CD's. The risky asset does not have to be a single stock, it could be a portfolio of gov bonds or CDs.

Riskless Borrowing and Lending

- Since we have a risk free asset then the standard deviation of the asset is zero. Also the covariance will also be zero.
- Therefore:

$$\sigma_P = w \times \sigma_S$$

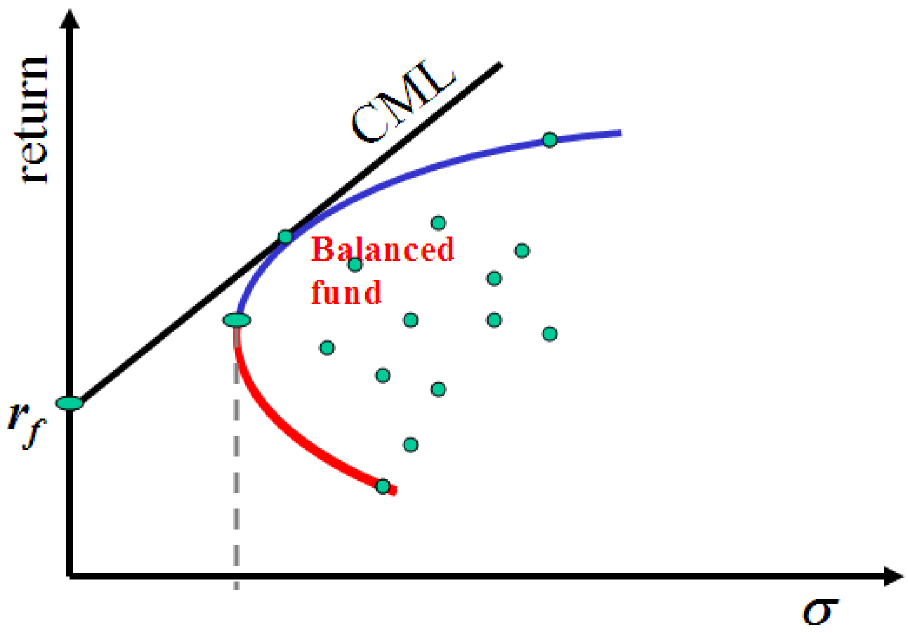
- Combining the two we have:

$$E\{r_P\} = r_f + (\sigma_P / \sigma_S) \times [E\{r_S\} - r_f]$$

Riskless Borrowing and Lending

- This is the equation of a straight line.
- An investor can combine the riskfree asset with any risky asset in the opportunity set.
  - However, the line that is tangent to the efficient set of risky assets provide investors with the highest return at any given standard deviation.

Riskless Borrowing and Lending



Market Equilibrium

- With the capital market line identified, all investors choose a point along the line—some combination of the risk-free asset and the market portfolio *M*. In a world with homogeneous expectations, *M* is the same for all investors.
- *The Separation Property* states that the market portfolio, *M*, is the same for all investors—they can *separate* their risk aversion from their choice of the market portfolio.

Market Equilibrium

- The separation property implies that portfolio choice can be separated into two tasks:

* (1) determine the optimal risky portfolio, and
* (2) selecting a point on the CML.

The Capital Asset Pricing Model

- So with the additional assumptions we can complete the development of the CAPM
  * Risk less Borrowing And Lending
  * Homogeneous Expectations
- Expected return on an individual security:

$$\bar{R}_i = R_F + \beta_i \times (\underbrace{\bar{R}_M - R_F}_{\text{Market Risk Premium}})$$

Definition of Risk When Investors Hold the Market Portfolio

- Researchers have shown that the best measure of the risk of a security in a large portfolio is the *beta* (β)of the security.
- Beta measures the responsiveness of a security to movements in the market portfolio.

$$\beta_i = \frac{Cov(R_{i,} R_M)}{\sigma^2(R_M)}$$

- Clearly, your estimate of beta will depend upon your choice of a proxy for the market portfolio.

Uses for CAPM

- Cost of capital estimation
- Portfolio performance
- Event-study analysis

Estimator for Beta

- Usually run regression of the stocks realized returns verses the corresponding realized market returns in excess of the risk-free rate
  - Often monthly for five years
  - Market portfolio often taken as the S&P 500
  - The risk free rate is a t-bill rate

Empirical Tests of CAPM

- P/E ratio effect (Basu 1977)
- Market capitalization (Basu 1981)
- Book to market value (F & F 1992, 1993)