Economic Markets

We want to work towards a theory of market equilibrium. We already have a pretty good idea of what that is, the prices and quantity in a market are going to be set by supply and demand. So to reach this point we need to understand:

1) Market Equilibrium (class 9)

2) DEMAND (Consumer Behavior) (class 7)

3) SUPPLY (Theory of the firm) (class 8)

We will also look at some markets that do not work the way we think they should. These would be monopolies and duopolies. In those markets the idea of supply will be corrupted in some way. There will be different decisions faced by someone in a monopoly as opposed to someone in a competitive market. (this will be another class)

The Theory of Consumer Behavior

We'll be considering these topics.

Overview

- Utility, how people order choices. Which choice gives more utility, a way to order consumption. An idea of the satisfaction I receive, the benefit.
- Basic Concepts in terms of utility. Simple indifference curves.
- Maximization of Utility, what level of each good, given by budget constraint, is going to give me the greatest utility? Dependent on income level and prices of goods. Find optimum level.
- Demand Functions, we can find solutions given any price and income.
- Elasticity's, if the price changes how much more or less are we going to demand of that particular good? Given a proportional change at a particular price what is the change in the amount of good we demand. This leads to an idea of luxury goods and necessary goods. Also cross elasticity. If the price changes for good 1, how much does that effect our demand for good two? Remember we only have a fixed amount of dollars to spend.

- Generalization
- I have used frequently the following classic text
  - Microeconomic Theory, Henderson and Quandt

In the following developments we will be considering a two good world. We will see how these concepts are very general.

Axioms of Rational Choice

- **Completeness**, given two choices an individual has to be able to say …
  - If A and B are any two situations, an individual can always specify exactly one of these possibilities:
    - A is preferred to B
    - B is preferred to A
    - A and B are equally attractive
- **Transitivity**
  - If A is preferred to B, and B is preferred to C, then A is preferred to C
  - Assumes that the individual's choices are internally consistent
- **Continuity**
  - If A is preferred to B, then situations suitably "close to" A must also be preferred to B (there are no jumps, the choices in between for a nice orderly line)
  - Used to analyze individuals' responses to relatively small changes in income and prices

To construct an idea of utility we need some idea of what we mean by rational choice. There are different ways of framing these but these axioms seem to fit with the way we see the world (although this is not a complete list).

We need a way of comparing two situations in order to construct an idea of how much we value those two situations.

Given these assumptions economist can construct Utility Functions.

All this means is people are able to rank, place in order, any given choices. Rank from most desirerable to least desirerable.

Utility

- Given these assumptions, it is possible to show that people are able to rank in order all possible situations from least desirable to most
- Economists call this ranking utility
  - If A is preferred to B, then the utility assigned to A exceeds the utility assigned to B
    U(A) > U(B)

The Utility of A or B or any choice is a number. We can place a value on the preference and if A is preferred to B then the utility of a, U(A) is greater than the utility of b, U(B).

Bear in mind this is an ordinal ranking. We cannot say that something is twice as preferred. It is only a ordering system, the distance between the two values doesn't tell you very much.

Given these three axioms we can rank things and construct a utility curve for an individual.

Basic Concepts

- Utility Functions
  - Two commodity case
  - U = U(x, y)
  - x is the **quantity** of commodity x consumed,…
  - It is assumed that the function U is continuous and has continuous first and second partial derivatives (which exist)
  - It is assumed that the first partial derivatives are strictly positive
    - Consumer will always desire more of each commodity

The utility if going to be a function of the consumption of two goods. If we can say how much of an item we have we can assign a value to that consumption choice.

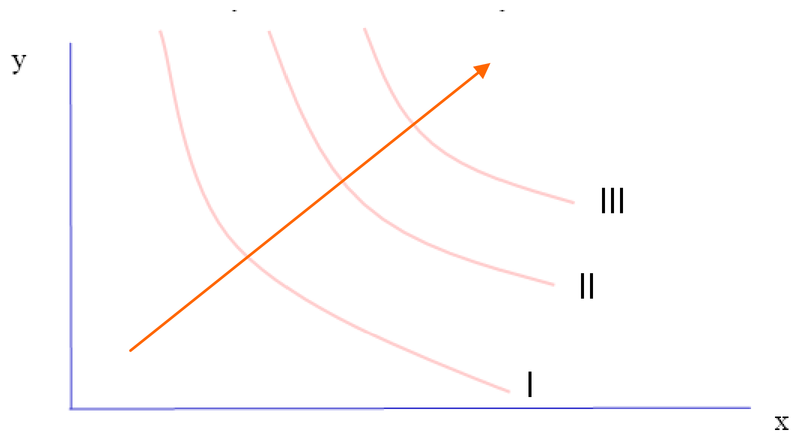
If I increase one of the variables then the utility should increase.

Once we get to extremes then increased consumption of a particular good doesn't necessarily increase our well being (utility). But we will not be considering these realms.

Basic Concepts, cont'd

❑ Indifference Curves

* Defined by the different combinations of the two commodities that yield the same utility



❑ Higher utility is found the further the indifference curve is from the origin

❑ Indifference curves do not intersect

The axis's represent amounts of good Y and good X. The utility curves (indifference curves) here are the curves where the utility remains constant for the given values of x and y. So if we plot the value of utility U(x,y) for any values or x and y, on the lines above the utility will be constant.

If I can receive more of both x and y my utility is going to increase (for instance from utility curve I to II).

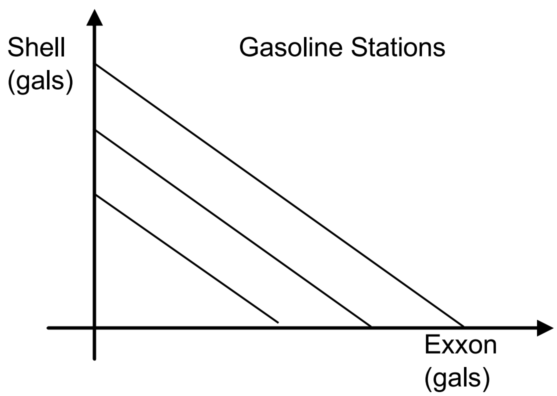
As we get further away from the origin (move toward the northeast) the utility is increased. This is a result of the assumption that the first partial derivative wrt any of the variables is increasing. As I increase the amount of goods I consume I assume my utility goes up. This means that $U_I(xy) < U_{II}(xy) < U_{III}(xy)$.

Called indifference curves because we are indifferent, in terms of happiness and utility, to the various points on the curve.

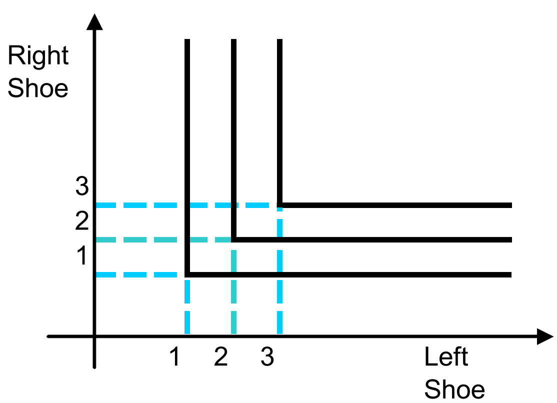
Ex. Hotels: y = in room services, x = out of room services
The hotels will construct using surveys how people feel about different combinations of in room and out of room services. Then they look at there competitors and where they are on these curves they have constructed (based on the services the competitor hotels offer). Now the hotel has a certain budget to spend to improve services, either in room, out of room, or both. They would construct models of different curves based on the changes they could make. They will construct a package which fits within the budget and

brings them to a higher indifference curve than their competitors. In this way they know that a particular combination of improvements is going to be the most beneficial. Tells them they can charge more for the rooms because they have the services customers will pay for.

Example



Notice that these are perfectly straight lines. This is because of tank capacity for one thing but also because we are indifference between gas from Shell and gas from Exxon. As long as the price is relatively close these are substitutes.



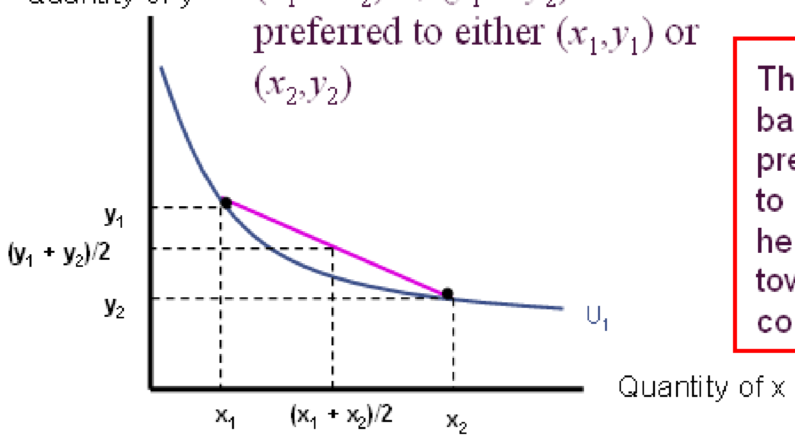
Example

Right and left shoe. The indifference curves indicate that we cannot reach a new level of satisfaction by having more than one right shoe and only one left shoe, there is no increase in utility. To get to a greater level of satisfaction (utility) I need more of each, another right and another left shoe.

This is an example complements. I need both to increase my utility. Here we are seeing that if I do not get the exact mix that I need then increasing one or the other does not increase my utility. Will stay at a given sweet spot unless I can improve on both. Hotdogs and buns, hamburgers and ketchup would be other examples.

❑ The curves are convex to the origin

If the indifference curve is convex, then the combination $(x_1 + x_2)/2$, $(y_1 + y_2)/2$ will be preferred to either (x_1, y_1) or (x_2, y_2)



This implies that "well-balanced" bundles are preferred to bundles that are heavily weighted toward one commodity

We usually assume that the curves are convex to the origin. Any two points on the curve joined by a straight line, convex to the origin means that the straight line is never any closer to the origin than the indifference curve.

This implies that a well balanced bundle of goods is always preferred to having an extreme of one good and only a little of the other. A bundle is a combination of the two goods. The graph of ½ the Y range and ½ the X range is showing that utility of the balanced bundle is greater than the indifference curve. Both of these points lead to the conclusion:

IF I JOIN TWO POINTS ON THE INDIFFERENCE CURVE THE RESULT WILL ALWAYS GIVE ME HIGHER INDIFFERENCE.

Joining two points will never give less utility (given the framework of these assumptions).

WE ARE SEEING HERE THAT PEOPLE DO NOT LIKE EXTREMES, GIVE A CHOICE OF TWO PARTICULAR GOODS THEY WILL ALWAYS PREFER A BALANCED MIXTURE.

NOTATION

The following notation will be use and all mean the same.

$$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy$$
$$= U_x dx \ + \ U_y dy$$
$$= U_1 dx \ + \ U_2 dy$$

Basic Concepts, cont'd

- ❑ Rate of Commodity Substitution
  - * The total differential of the utility function is: $dU = U_x\, dx + U_y\, dy$
  - * Where U_x and U_y are the partial derivative of U w.r.t. x and y
  - * Suppose we fix an indifference curve and decrease x by a small amount and compensate by increasing y by a small amount

Rate of Commodity refers to the idea that if I travel along the indifference curve at any particular point how much am I giving up of one good to receive another good? If I fix my utility, move down that curve, what rate am I exchanging one good for another in order to remain at the same utility? Text books will call this Marginal Rate Commodity Substitution (MRCS).

Rate of Commodity Substitution, cont'd

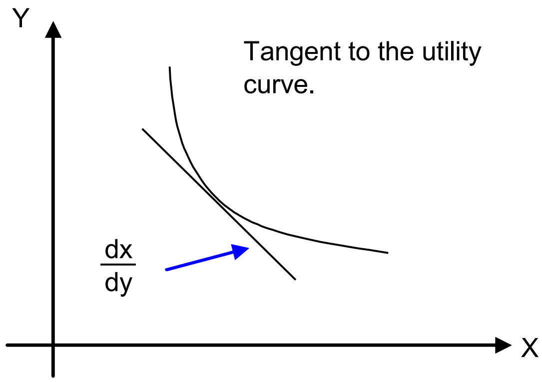
- ❑ Notice that **U_x dx** is approximately the **loss in utility** by decreasing **x** by a small amount and **U_y dy** is approximately the **gain in utility** by increasing **y** by a small amount
- ❑ **Setting dU = 0 yields** (how much to change x in order to compensate for a change in y)

  $U_x\, dx + U_y\, dy = 0$, so $U_x\, dx = -U_y\, dy$, and …

$$-\frac{dy}{dx} = \frac{U_x}{U_y}$$

This is why it is called **Rate** Commodity Substitution. If I want to maintain the same utility, if I give up some Y, how much extra X do I need (and visa-versa). We are moving down one of the indifference curves. We set the small change in utility to 0 in order to stay on the same indifference curve.

Notice U_x dx is approximately the loss in utility by decreasing x by a small amount. U_y dy is approximately the gain in utility by increasing y by a small amount. I've set the change in utility to 0 (dU = 0) so I'm moving along an indifference curve.



If I want to change one good for another how much do I need to change it in order to maintain the same level of utility?

dy/dx gives the slope of the curve. If I am moving down an indifference curve, the slope tells me how much I have to compensate for a loss in x by increasing y to stay on the same indifference curve.

Example: Utility and the RCS

- Suppose an individual's preferences for **beer (y)** and **pizzas (x)** can be represented by

$$\text{utility} = 20 = \sqrt{x \cdot y}$$

This form of the utility function is called Cobb-Dobles (?)

Suppose we are interested in the indifference curve where my utility is 20. Utility is a number, it is some measure of my world view. What I am interested in is the mixture of x and y that give me a utility of 20.

A simple way or expressing the Rate of Commodity Substitution there are a number of ways but probably the simplest is to express y as a function of x and then take the derivative.

- Solving for y, we get y = 400/x
- Solving for RCS = -dy/dx:

$$\text{RCS} = -dy/dx = 400/x^2$$ ←RCS is the derivative

Started by setting our utility function to the value we are interested in, 20, solving for y in terms of x, and taking the derivative which we call RCS.

So now we know that we need to divide 400 by X in order to maintain a utility of 20. This describes the utility curve where my utility is 20. Given any amount of pizza (x) I can tell you how much beer (y) I need to generate that level of utility (20).

Utility and the RCS

$$RCS = -dy/dx = 400/x^2$$

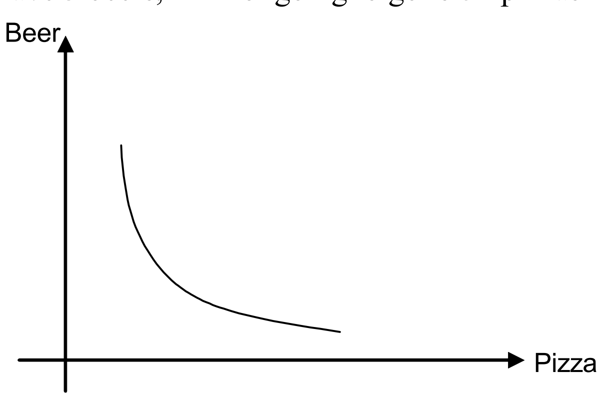
- Note that as x rises, RCS falls
  - when x = 5, RCS = 16
  - when x = 20, RCS = 1

$$\text{Rate of Commodity Substitution} = RCS = \frac{-dy}{dx}$$

In our example $y = 400/x$ so $RCS = -dy/dx = 400/x^2$

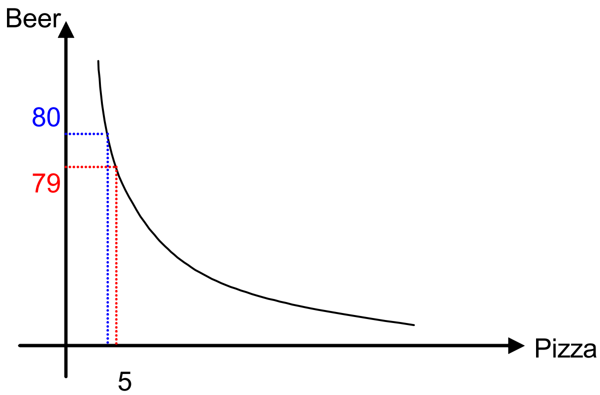
This tells me how much beer I need to give up for pizza in order to maintain the utility of 20.

Notice that as x increases the rate of commodity substitution falls. What does this mean? This is saying "as I get more and more pizza compared to beer, I'm less willing to exchange more to add to my pizza and give up beer. If I'm sitting at 50 pizzas and I only have 3 beers, I'm not going to go to 51 pizzas to give up one beer. It's not going to give me any benefit. It's a function of the indifference curve being sloped in this way. As I move up this curve I get less and less benefit from adding more of y at the expense of x. I have to get far more in exchange. At the other extreme, if I've got 400 beers and only 2 pizzas I'm not going to give up very much pizza for one more beer. It's not going to give me much more value.



When x = 5, RCS = 16, when I have 5 pizzas I'm willing to exchange beer for pizza at a rate of 16.

When x = 20, RCS = 1, at 20 pizzas the rate of commodity substitution has fallen to 1. I'm not willing to give up my beer in order to add pizza, I'm already saturated with pizzas.



Consider x = 5, then y = 400/5 = 80 in order to keep our satisfaction at 20. Suppose I give up one beer, how much more pizza do I need to stay at U=20? If I give up one beer my y goes from 80 to 79 and my x goes from 5 to 400/79 = 5.06. So I've given up one beer and not gotten very much pizza. But we are not surprised because we are in a high slope area of the curve, in this region if I give up a beer I don't expect much pizza back.

Previously we had calculated that at x=5 the RCS is 16, steep slope. This is the graphical representation. The small change in y is 1 beer. The small change in x = 1/.06 = 16.6.

On a fixed indifference curve, at a fixed level of utility, how much do I have to exchange of one good for another to remain at the same level of utility.

BE AWARE THAT THE RATE OF COMMODITY SUBSTITUTION AS I GO DOWN THE SAME INDIFFERENCE CURVE. THIS IS JUST THE SLOPE CHANGING AT DIFFERENT POINTS IN THE FUNCTION.

We can see that in this example, by the time I get to 20 pizzas I'm giving them up 1 for 1 (from x=1 gives RCS = $400/20^2$= 1).

As a result of our function being convex wrt to origin, the rate of commodity substitution is diminishing in y. As I come down the curve from high levels of y to low levels of y the slope becomes shallower.

$$\text{Verify RCS} = \frac{\text{-dy}}{\text{dx}} = \frac{U_x}{U_y}$$

$$U_x = \frac{\partial}{\partial x}\sqrt{xy} = \frac{1}{2}\sqrt{\frac{y}{x}}$$

$$U_y = \frac{\partial}{\partial y}\sqrt{xy} = \frac{1}{2}\sqrt{\frac{x}{y}}$$

$$\frac{U_x}{U_y} = \frac{\frac{1}{2}\sqrt{\frac{y}{x}}}{\frac{1}{2}\sqrt{\frac{x}{y}}} = \frac{\sqrt{y}}{\sqrt{x}} \bullet \frac{\sqrt{y}}{\sqrt{x}} = \frac{y}{x}$$

$$\text{substitute } y = \frac{400}{x} \text{ to get } \frac{U_x}{U_y} = \frac{400}{x^2}$$

Maximization of Utility

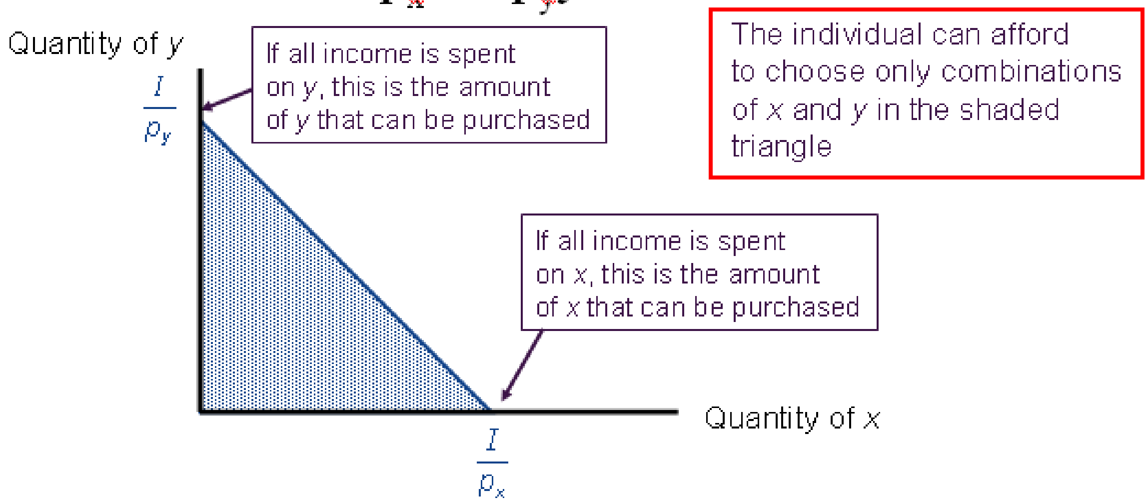
- To **maximize utility**, given a fixed amount of income to spend, an individual will buy the goods and services:
  - **That exhaust her total income**
  - **For which the rate of trade-off between any goods (the RCS) is equal to the rate at which goods can be traded for one another in the marketplace**
- Assume that the individual's RCS = 1
  - Willing to trade one unit of x for one unit of y
- Suppose the price of x = $2 and the price of y = $1
- The individual can be made better off
  - trade 1 unit of x for 2 units of y in the marketplace

If my expenditure is not constrained then I can never maximize my utility. If someone just continues to give me funds my utility is just going to increase. Therefore **to maximize my utility I will have to make the assumption that I have a certain budget over a certain period of time**. Given that amount of money what mixture of the two goods will I choose that maximizes my well being?

In our model there is no carry over of money from one period to another, all income is spent. Now how do we find the best mixture given our monetary budget? It's going to be when the trade off of the two goods on our indifference curve is equal to the difference in the price of those two goods.

- Assume that an individual has I dollars to allocate between good x and good y

$$p_x x + p_y y \leq I$$



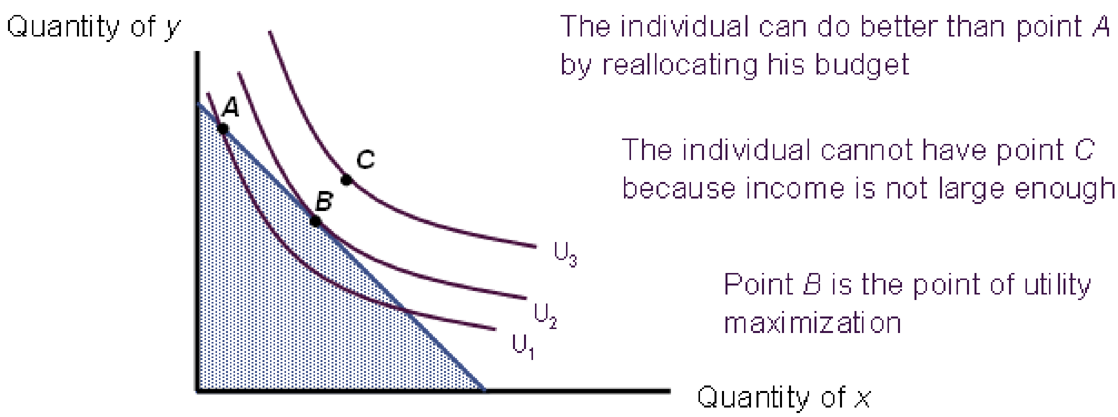
We could spend all of our money on one of the goods, these are the line end points where income I is over price of that commodity (p_x or p_y). Or I could spend my money on any combination of goods between the two end points. How will we maximize our utility?

We introduce an indifference curve…

First-Order Conditions for a Maximum

- We can add the individual's utility map to show the utility-maximization process

Py = price of 1 unit of good y.
Px = price of 1 unit of good x.



Consider point C on U_3. This is beyond the range our budget can achieve, we cannot reach this level. There is no mixture of x and y that I can afford which will get me to that level.

We can reach a number of points on curve U_1 (A for instance) but these are sub-optimal. Point B represents the tangential intersection of the budget constraint and the utility curve U_2. This is the best utility that I can achieve given that mixture of prices and that level of income. All the maximization problem does if find the indifference curve. That just touches the budget constraint. Can't go any further because not enough money. Don't want to drop down because I could consume more and be better off.

$$\text{At point B the slope of the curve is RCS} = \frac{-dy}{dx}$$

At this point or any point on the straight line the slope is the ratio of the two prices, $\frac{p_y}{p_x}$.

When I maximize my utility the rate at which I am willing to give up the goods is equal to the price ratio between the two goods.

THE RATE OF COMMODITY SUBSTITUTION IS EQUAL TO NEGATIVE THE RATIO OF THE PRICES

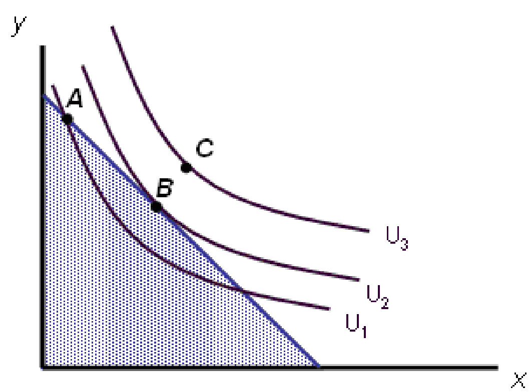
TRADE OFF BETWEEN ANY TWO GOODS, THE RCS IS EQUAL TO THE RATE AT WHICH THE GOODS CAN BE TRADED FOR ONE ANOTHER. THAT IS THE RATIO OF THE PRICES.

Suppose I have an individual at a particular mix of x and y (the rate of commodity substitution changes as you move along an indifference curve) and at this particular point the RCS is 1 to 1. Remember, commodity substitution is measured in change in the number of goods, no dollar value associated.

Giving up one good y for one good x, RCS = 1.

Now suppose the price of x is $2 and the price of y is $1. I only have a certain amount to spend. Therefore if I trade one unit of x (sell one unit of x or not purchase one unit of x) I'm better off by $2. I can take that two dollars and buy two units of y. That is going to make me strictly better off. Because at this point I'm willing to trade 1 for 1, but because of the market place and the price I can actually give up one and receive two. It all comes down to the fact that at this point the rate of substitution is equal to the ratio of the prices.

If I can exchange better then I can receive more and reinvest it in the other commodity.

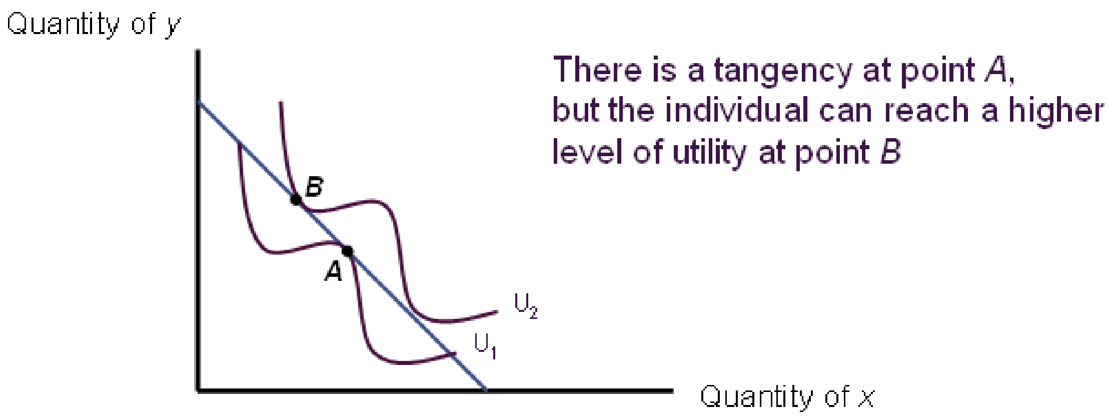


The shaded area represents combinations that I can possibly consume. In the sense of efficient markets we can be in the shaded area if we wish. Because I cannot carry forward money I will not be in the shaded area, I will be on the boundary, because there is no point in throwing away money (since I cannot carry it forward). Away from the boundary is in effect just burning excess cash.

Since indifference curves increase as you move northeast (away from the origin) and they are convex to the origin and we are always on the boundary of our spending line, we will always seek point B where spending is optimal to utility. (the curves cannot be wiggly like below). This curve implies that if there is no budget constraint we would just keep increasing our utility level.

Second-Order Conditions for a Maximum

- The tangency rule is only a necessary condition
  - we need RCS to be diminishing



Would be better off at point B because higher utility level. This is why we must assume our utility curves are convex to the origin. It can be proven that they are.

Notice that as long as our rate of commodity substitution is diminishing then we are always safe (?). $400/x^2$ is certainly diminishing.

The Lagrangian

- The **individual's objective is to maximize utility** = U(x, y) **subject to the budget constraint:** $I = p_x x + p_y y$

How do we find the optimal point? There is a way of maximizing a function given a constraint. There are several requirements to using this method but basically as long as our RCS is diminishing were basically safe. (skipping some possible solutions).

What we will do is maximize U(x,y) subject to the constraint that the answer has to lie on the budget line. We will define this line as Income equals the price of good x times the number of units of good x plus the price of good y times the number of units of good y.

Budget Constraint: $I = p_x x + p_y y$

When x is zero $y = \frac{I}{p_y}$ and when y is zero $x = \frac{I}{p_x}$. The slope of this line is $\frac{-p_x}{p_y}$.

When RCS = $\frac{-p_x}{p_y}$ we are optimal. To solve this type of constraint optimization we use the Lagrangian method…

- Set up the Lagrangian: $\mathbf{L = U(x, y) + \lambda(I - p_x x - p_y y)}$

Works because the budgetary term is always set to zero and we solve for the best value of utility we can find along that line.

The Lagrangian, cont'd

- First-order conditions for an interior maximum:

$$\partial L/\partial x = \partial U/\partial x - \lambda p_x = 0$$
$$\partial L/\partial y = \partial U/\partial y - \lambda p_y = 0$$
$$\partial L/\partial \lambda = I - p_x x - p_y y = 0$$

Now we take the partial derivatives wrt x, y, and lambda setting each equal to zero. Notice that when taking the derivative wrt lambda the budget expression is treated as a constant.

Implications of First-Order Conditions

- For the two goods, (using the above equations)

$$\frac{\partial U/\partial x}{\partial U/\partial y} = \frac{p_x}{p_y}$$

This is telling us that the Rate of Commodity Substitution (RCS) is equal to the slope of the budget constraint line. The rate at which I'm willing to give up x for y is equal to the ratio of the price of x to the price of y. I'm giving up units of goods x for units of goods y. The optimal point is when the rate of exchange is equal to the ratio of the market prices.

If I can give up two slices of pizza for one glass of beer and be indifferent (the price of each is one-to-one) then I'm going to do it because I can earn \$2 and buy more beer. The solution will be found where this is the case.

- This implies that at the optimal allocation of income

$$RCS\ (x \text{ for } y) = \frac{p_x}{p_y}$$

WHAT HAPPENED TO THE NEGATIVE SIGN ?

Interpreting the **Lagrangian Multiplier**

$$\lambda = \frac{\partial U/\partial x}{p_x} = \frac{\partial U/\partial y}{p_y}$$

Lambda is described as the shadow price of the constraint. Lambda represents how much my utility improves if I can achieve another dollar of income. Tells me how much better off I am if I receive additional income to spend.

Lambda is telling me if I can shift my budget constraint by a small amount how much my utility improves. λ is the marginal utility of an extra dollar of consumption expenditure.

- λ is the marginal utility of an extra dollar of consumption expenditure
  - the marginal utility of income

Lambda does not measure a dollar increase in my utility. It tells me how much my well being increases for a given dollar change in my income. **Utility is not measured in dollars. Utility is just a sense of well being, an ordering.**

Example

- Suppose we have a utility function of the form:

  $U(x, y) = xy$
- Suppose that $p_x = \$2$ and $p_y = \$5$
- The consumer's income is \$100
  - Therefore the **budget constraint** is given by:

    $100 - 2x - 5y = 0$ ← zero means we are going to spend everything

Price of good x is \$2 and price of good y is \$5. Income for period is \$100.

- Set up the Lagrangian:

$$L = U(x, y) + \lambda(I - p_x x - p_y y)$$
$$= xy + \lambda(100 - 2x - 5y)$$

- First-order conditions for an interior maximum:

$$\partial L/\partial x = \partial U/\partial x - \lambda p_x = y - 2\lambda = 0$$
$$\partial L/\partial y = \partial U/\partial y - \lambda p_y = x - 5\lambda = 0$$
$$\partial L/\partial \lambda = I - p_x x - p_y y = 100 - 2x - 5y = 0$$

Solve these three equations with three variables for the value of x, y, and lambda.

$$y - 2\lambda = 0 \quad \text{--->} \quad \lambda = \frac{y}{2} = \frac{10}{2} = 5$$

$$x - 5\lambda = 0 \quad \text{--->} \quad x = \frac{5y}{2} = \frac{50}{2} = 25$$

$$-2x - 5y = -100 \quad \text{--->} \quad -5y-5y=-100 \quad \text{--->} \quad y=10$$

$Y = 10$ $x = 25$ $\lambda = 5$

This is how much of each good (at these prices and this budget constraint) I want to consume. λ is telling us how much the utility increases if our income is increased by \$1, this is the shadow price of income.

What is our level of utility at this point? U(25, 10) = (25)(10) = 250. Now suppose we increase our budget by $1 to $101. Now completely redo the solution using the new budget of $101. Will find that:

y = 10.1 x = 25.25 xy = (10.1)(25.25) = 255.025

The new utility is approximately $5 more than the original utility, this confirms λ solution above (because λ is only approximate).

Example, cont'd

- Solve to get x = 25, y = 10 and λ = 5
- Thus, if the budget constraint can be increased by $1, then utility will increase by approximately λ = 5
- Check to see:
  - U with budget set at $100 is xy = 25 * 10 = 250
- Solve again with budget set at $101 gives
  - x = 101/4, y = 101/10 and xy = 255.025

BACK TO ORIGINAL SOLUTION:

Y = 10 x = 25 λ = 5 xy = 250

Express as y = 250 / x then $\frac{dy}{dx} = \frac{-250}{x^2}$ plug in the solution for x to get

$$\frac{-dy}{dx} = \frac{250}{x^2} = \frac{250}{25^2} = \frac{250}{625} = \frac{10}{25} = \text{RCS} = \frac{2}{5} = \frac{P_x}{P_y}$$

This says I'm willing to trade x for y at a 2/5 ratio which also ties up with the value in the market place. This is saying that I can be no better off moving down the indifference curve. The ratio I'm willing to give up goods to the market place equals the ratio of the market place price for the goods. This leads into demand functions…

We want to construct the market places supply versus demand. We've built an idea of to decide how much the consumer is willing to consume of a bundle of goods and where the utility is maximized. We will take the solutions to that and turn the process around. We will construct demand functions.

Demand Functions

- Ordinary Demand Functions
  - An individual's demand for *x* depends on preferences, all prices, and income:

$$x^* = x(p_x, p_y, I)$$

  - It may be convenient to graph the individual's demand for *x* assuming that income and the price of *y* (p_y) are held constant

The demand function for a particular good is how much of that good we want to consume given the price of that good. We can see by the arguments we are using in the utility maximization that the amount that we actually consume of the good will be dependent on our income, the price of other goods in the market place, and on the price of our particular good that we are looking to consume. How does the demand for a good vary as it's price changes assuming everything else remains constant?

We often assume when crafting these types of things that as we look at the demand for x given the price of x holding the price of y and the income that we have constant. See how our demand changes. Create a demand curve for good x against it's price holding everything else constant. (This is our goal).

Continuing on with our example …

Demand Function Example

- Suppose we have a utility function of the form: U(x, y) = xy
- Budget Constraint: I – pxx – pyy
- Once again set up the Lagrangian:

$$L = U(x, y) + \lambda(I - p_x x - p_y y)$$
$$= xy + \lambda(I - p_x x - p_y y)$$

  - Notice we are not assuming any value of p_x, p_y or I in this example
- First-order conditions for an interior maximum:

$$\partial L/\partial x = \partial U/\partial x - \lambda p_x = y - p_x\lambda = 0$$
$$\partial L/\partial y = \partial U/\partial y - \lambda p_y = x - p_y\lambda = 0$$
$$\partial L/\partial \lambda = I - p_x x - p_y y = 0$$

- Solving for x and y gives the demand functions

Solve for x and y the solutions are the demand curves.

$y = p_x\lambda$ $x = p_y\lambda$ use $I - p_x x - p_y y = 0$ to get the solution …

$$x = I/2p_x \quad \text{and} \quad y = I/2p_y$$

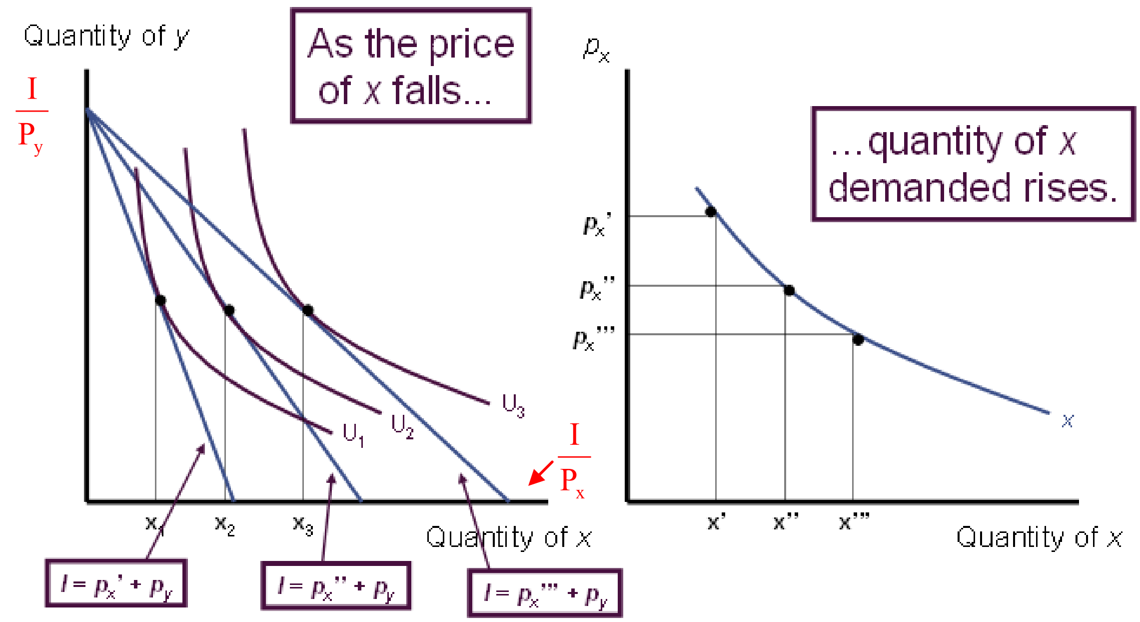
- Notice these demand functions are the special case in which the demand for each commodity depends only upon its own price and income
- It is easy to show that for any utility if all prices and income change in the same proportion, the quantities demanded remain unchanged

Given income and price we can tell what demand is.

This is a very special case. The demand for x is only dependent on the price of x and our income level, same is true for y, only dependent on income level and price of y good. Generally this will not be the case, only have this because we used the simple utility function U(x, y) = xy. If we add a little complexity to the utility function the x and y demand will be dependent on the price of the other good as well. But keep in mind that when we are plotting demand curves we are assuming that those other prices are constant.

Another point, it's actually very easy to show, and this is true for any utility, if I increase my income and I increase my prices by the same ratio I will still get the same solutions. Still will consume the same amount of goods.

The Individual's Demand Curve



Given the budget constraint and the shape of the indifference curve and utility level above we can get a very general answer. The amount of goods x I will consume at a certain level. Given a certain price I will consume so much of good x. Now we will change the price of x, we will decrease the price of x. If we do so what will happen to the budget constraint? Consider the endpoints of the budget constraint lines, I/Py and I/Px. If I decrease my price the budget constraint line becomes shallower. This says that if I only consume x I can consume more of it (because the price has come down). If my price comes down from 10 to 5, if all I consume is x I can consume more for a given level of income. (So the graph on the left is showing the price of x coming down?)

So the line becomes shallower. Now I do the maximization problem again. I receive another utility and I'm better off because the price of good x has fallen, I don't have to spend so much on good x. I can afford to spend more on good y or consume more on good x or both. But what physically happens? As the price falls I'm almost certainly going to consume more of x. Now we reduce the price of x even further. Budget constraint line becomes shallower, utility comes to some new level, the amount of x I consume will increase. Therefore the amount I consume at the lowest price is even higher. As the price of a good falls you end up consuming more and more of the good.

So we've seen that as the price of x falls the number of units that I consume increases.

The Individual's Demand Curve

- An individual demand curve shows the relationship between the price of a good and the quantity of that good purchased by an individual assuming that all other determinants of demand are held constant

The individuals demand curve will show the relationship between the price of the good and how much good I'm going to purchase if I hold everything else constant.

I solve my capitalization problem, I lock in on that particular good, I solve for that particular good, the amount that I consume is going to be a function of my income and the price of all other goods. The demand curve describes that relationship if I hold the income and the other prices constant.

Shifts in the Demand Curve

- Three factors are held constant when a demand curve is derived
  - income
  - prices of other goods (p_y)
  - the individual's preferences
- If any of these factors change, the demand curve will shift to a new position
- A movement along a given demand curve is caused by a change in the price of the good
  - a change in quantity demanded
- A shift in the demand curve is caused by changes in income, prices of other goods, or preferences
  - a change in demand curve

Demand curves will shift, they will change. If my income changes my demand curve will change. Remember, I'm holding my income constant. If I increase my income it's going to shift my demand curve.

Prices of the other goods, if I don't hold that constant I'm going to end up with new demand curves.

Individuals preferences, if I change my utility function then my demand curve is going to change.

Any of these factors will shift the demand curve for a particular good for a particular consumer. Moving along the demand curve is caused by a change in the price of the good. Price goes up, quantity falls. Price goes down, quantity will go up.

Shift in the demand curve, actual change in the entire curve, will occur because of income, prices of the other goods, or preferences. Holding all things constant and I get changes in price/good relationships, if I vary then my curves actually change.

Demand Elasticities

- The own price elasticity of demand for x (which we will call ε_{xx}) is defined as the proportionate rate of change amount consumed x, divided by the proportionate rate of change of its own price, holding income and the other price constant

$$\varepsilon_{xx} = \frac{p_x}{x}\frac{\partial x}{\partial p_x}$$

All we are considering here is how much the demand for a good changes given a change in prices. The simplest one to think of is the own price elasticity. Given a change in the price of a good, how much does the demand for that good change? How much do I move along the demand curve? We define this as eplison sub xx. **What this means is the change in the amount of a good that I demand given a change in the price of that good**.

ε change in the amount of a good that I demand, given a change in the price of that good

I change the price of Px by a small amount, a tiny little piece of x. Ex., the price is $5 I change it to $5.01. The proportionate change is the small amount I'm going to change divided by current price. If the price is a dollar and I increase the price by .01, then my proportionate change is a 1% increase. That's going to lead to a change in the goods that I want to consume. How much I want to consume of x.

What's the proportionate change? This will lead to a change in good x (in the literature, the good is the capital letter, the amount consumed is the small letter). So a good currently at x is changed by a small amount Δx. Price goes up by 1%, what is the effect on the amount demanded?

Proportionate Change = $\frac{\Delta x}{x}$

The own price elasticity is the percentage change divided by the percentage change. (?)

This is approximately equal to $\varepsilon_{xx} \approx \dfrac{\text{Percentage change in x}}{\text{Percentage change in P}_x}$.

If the prices go up by 1% what happens to the amount of good that I receive? It's going to be negative. Demand is going to fall. Elasticity tells us what that ratio is.

For loanable funds interest is the price of loanable funds. This is saying that interest is inelastic if the amount that the government wishes to barrow doesn't change very much given a change in interest rates.

If interest rates go up by 100%, say from 5% to 10%, the government is not going to see any significant change in the amount of funds it demands. The epsilon ratio will be close to zero. The government demand for loanable funds is inelastic.

Physically what is going on is that x is the demand function for the amount of goods x that we are going to consume, you calculate the partial derivative of x wrt it's price. (example below)

Two stage process, find the demand functions and then calculate the partial derivatives.

Own Price Elasticity of Demand

- The own price elasticity of demand is almost always negative
- The size of the elasticity is important
  - $\varepsilon_{xx} < -1$, demand is **elastic** (luxuries)
  - $\varepsilon_{xx} > -1$, demand is **inelastic** (necessities)
  - $\varepsilon_{xx} = -1$, demand is **unit elastic**

If the size of the elasticity is <-1, demand is called elastic. Steep Slope. Given a change in prices there is a larger change in the goods consumed. A 1% change in prices is going to lead to a > 1% decrease in the goods I consume. Slope is steep, less than -1. These items are called luxuries. As the price of luxuries goes up the demand falls sharply.

If the size of the elasticity is > -1, demand is called inelastic. Remember, ZERO is greater than -1. So at the extreme there is no change in demand given a change in price. These things are called necessities. Will have to pay whatever the price, inelastic.

Right at -1 the demand is called unit elastic.

Back to our example …

Price Elasticity and Total Spending

- Total spending on x is equal to

  total spending = $p_x x$
- Using elasticity, we can determine how total spending changes when the price of x changes

$$\frac{\partial(p_x x)}{\partial p_x} = x + p_x \frac{\partial x}{\partial p_x} \qquad = x\left(1 + \frac{p_x}{x}\frac{\partial x}{\partial p_x}\right) = x(1+\varepsilon_{xx})$$

U = xy we have $x = \frac{I}{2P_x}$ Want to calculate it's Own Price Elasticity so what I need to calculate is first the rate of change of x wrt it's own price $\frac{\delta x}{\delta P_x} = \frac{-I}{2P_x}$. This is one of the factors in Own Price Elasticity. Therefore the Own Price Elasticity for x will be

$\varepsilon_{xx} = \frac{P_x}{x}\frac{\delta x}{\delta P_x} = \frac{P_x}{\frac{I}{2P_x}}\frac{-I}{2P^2{}_x} = -1$ in this case the Own Price Elasticity is UNIT ELASTIC.

What does this physically tell me? If the price falls by 1%, demand increases by 1%.

NOTE: THIS IS SPECIFIC TO U(X,Y) = XY.

Other utility functions will have other results!

Because the utility function is completely symmetrical I also know that Own Price Elasticity of y will also be -1. Look back to the demand functions for y, it was just $y=I/2P_y$. Redo the calculation wrt y, will get -1. Nothing special about x or y in this utility function.

EXAM **If given a symmetrical utility function and we are asked to calculate two Own Price Elasticity's, we only really have to do one, the other is symmetrical!**

Price Elasticity and Total Spending

- The sign of this derivative depends on whether ε_{xx} is greater or less than –1
  - $\varepsilon_{xx} > -1$, demand is inelastic and price and total spending move in the same direction
  - $\varepsilon_{xx} < -1$, demand is elastic and price and total spending move in opposite directions

If I change the price of a good, I'm not now interested in how the demand for that good changes, I want to know how much I spend on that good changes. We have said that as the price goes up the number of goods consumed declines. But **if the price doesn't go up fast enough**, the price times the number of goods, the actual dollar amount I spend, could go up and could go down. How much I actually physically spend on that particular good may go up or may go down depending on how quickly the price moving. **Is the price increasing quicker than the demand is falling?** This is where we will have to examine to find the tipping point.

It's very easy. If I lock in on the total spending, whatever the price and whatever the demand, remember these are solutions from our utility maximization, given my utility and given my income, this is the amount that I spend, and this is the number of goods I consume. Lock in on this, if I change the price how much does my total spending on good x change? That's what we want, the dollar amount we spent on x. If the price changes how am I moving on my demand curve?

Two stage derivative, $\frac{\delta}{\delta \mathrm{x}} P_x x = P_x \frac{\delta x}{\delta P_x} + x \frac{\delta P_x}{\delta P_x} = x + P_x \frac{\delta Px}{\delta Px}$

$$= x + P_x \frac{\delta x}{\delta P_x} = x + x \frac{P_x}{x} \frac{\delta x}{\delta P_x} = x(1 + \frac{P_x}{x} \frac{\delta x}{\delta P_x}) = x(1 + \varepsilon_{xx})$$

If I decrease the price is my dollar expenditure going to go up or go down?

If I decrease the price then the amount of goods I demand goes up, x is going to go up. If P_x goes down, x goes up. Change in dollar value $P_x x$, is that going to go up or down? There si no obvious way of knowing because ones going up and ones going down. This gives us the answer. If the elasticity < -1, then $(1 + \varepsilon_{xx})$ is negative (?).

Is the old price elasticity greater or less than -1? If it's greater than (1-1), demand is inelastic, they are necessities. If the price goes up, total spending goes up. Think about it like water, if the price of water goes up I still basically consume the same amount (the demand comes down a little bit, I might consume a little bit less but I will end up spending more in dollar terms). So therefore I end up spending more. Whereas for a luxury good, if the (err ?) of elasticity is less than -1, if the price goes up I consume so much less that I end up spending less in dollar terms.

The whole key is weather (1 + ε_{xx}) is greater than or less than -1.

In the case of our example the Own Price Elasticity is -1. What that told me is a 1% increase in price represents a 1% decrease in value to the consumer. So in our example the dollar amount we spend on x does not change. I just consume less to compensate for having to spend more. But I stay at the same dollar value.

Cross-Price Elasticity of Demand

- The cross-price elasticity of demand for y with respect to price p_x (which we will call ε_{yx}) is defined as the proportionate rate of change amount consumed y, divided by the proportionate rate of change in the price of x, holding income and the other price constant

$$\varepsilon_{yx} = \frac{p_x}{y} \frac{\partial y}{\partial p_x}$$

- Cross-price elasticities may be positive or negative
- Substitutes have positive cross price elasticities: Butter & Margarine
- Complements have negative cross price elasticities: DVD machines and the rental price of DVDs

Same idea, it's how much the demand for y changes if the price of x changes. Why would it, the price of y? Well we only have so much to spend, if the price of one good changes then it may have an impact on how much I spend on another good.

What we are looking at is how much does the amount of y, in percentage terms, change given the change in the price of x? if the price of x goes up 1%, what happens to the amount of y that I consume?

Whereas Own Price Elasticities are negative (price up means the number of goods I consume goes down), with Cross Price Elasticity who knows! I may consume more and I may consume less.

The connection comes from weather or not they are substitutes or are they compliments. Under these circumstances if the price of hamburgers goes up I am likely to consume less rolls, if the price of CDs goes up I might start using analog tape.

They can move in opposite directions dependent on weather they are compliments or substitutes.

For our particular case, the amount consumed of x = I/2P_x and the amount consumed of y = I/2P_y. Therefore if I take dx/dP_y it will be the same as dy/dP_x (these are partical derivatives) and they are both zero.

If the price in our model, where the utility function has the form xy, if the price of one good changes it has no impact on the other good.

We can take the analysis a step further. When we calculated this thing (?) we came to the conclusion that the dollar amount that I was going to spend on x would remain constant. Because the price elasticity was minus 1. It set this thing (?) equal to zero. So the dollar value that I spent on x remain constant. That is why this is zero. Sending the same dollar amount on x so therefore I still have the same dollar amount to spend on y. So therefore changing the price of x, in this example, does not change the value of y that I consume.

Cournot Aggregation

- Consider the budget constraint:

$$I = p_x x + p_y y$$

- Differentiating both sides by p_x yields:

$$\frac{\partial I}{\partial p_x} = p_x \frac{\partial x}{\partial p_x} + x + p_y \frac{\partial y}{\partial p_x} + y \frac{\partial p_y}{\partial p_x}$$

- Since we are holding income and the price of p_y constant by assumption then:

$$\frac{\partial I}{\partial p_x} = \frac{\partial p_y}{\partial p_x} = 0$$

- Therefore

$$p_x \frac{\partial x}{\partial p_x} + x + p_y \frac{\partial y}{\partial p_x} = 0$$

Cournot Aggregation , cont'd

- Multiplying all terms by p_x/I gives:

$$\left(\frac{p_x}{x}\frac{\partial x}{\partial p_x}\right)\left(\frac{p_x x}{I}\right)+x\frac{p_x}{I}+\left(\frac{p_x}{y}\frac{\partial y}{\partial p_x}\right)\left(\frac{p_y y}{I}\right)=0$$

- Or

$$\alpha_x\varepsilon_{xx}+\alpha_y\varepsilon_{yx}=-\alpha_x$$

- Where $\alpha_x = p_x x/I$ and $\alpha_y = p_y y/I$
  - The proportion of income spent on each good

Cournot Aggregation , cont'd

- The expression:
- Is described as the Cournot aggregation condition
  - Useful since it allows cross price elasticities to be calculated without construction demand functions
  - If $\varepsilon_{xx} = -1$ then $\varepsilon_{xy} = 0$
  - If $\varepsilon_{xx} < -1$ then $\varepsilon_{xy} > 0$
  - If $\varepsilon_{xx} > -1$ then $\varepsilon_{xy} < 0$

If the Own Price Elasticity is -1 then this relationship tells us I have minus a constant and the same minus constant on the other side. Therefore the cross price elasticity is zero. If the price elasticity is < -1, the some term is less than minus alpha, therefore we solve and end up with a positive value. Which implies that this thing is greater than 0.

Comes back down to this, if my dollar amount increases for a particular good, then the amount that I consume for the other good has to decrease to compensate. When does the dollar amount increase? When this is less than or greater than -1.

Understand why this is the case, why this is tied to this particular interpretation. Remember, we are on the same budget constraint line, if my dollar amount of 1 changes, goes down, the dollar amount of the other goes up. If the price is the same then the amount I consume goes up.

Income Elasticity of Demand

- Defined as the proportionate change in the purchases of a commodity relative to the proportionate change in income with prices held constant
- Can be positive or zero, but usually found to be negative

$$\eta_1 = \frac{I}{x}\frac{\partial x}{\partial I}$$

Generalization

- The optimal levels of $x_1, x_2, \ldots, x_n$ can be expressed as functions of all prices and income
- These can be expressed as n demand functions of the form:

$$x_1^* = x_1(p_1, p_2, \ldots, p_n, I)$$
$$x_2^* = x_2(p_1, p_2, \ldots, p_n, I)$$
$$\ldots$$
$$x_n^* = x_n(p_1, p_2, \ldots, p_n, I)$$

Conclusion

I can always do this for two variables if I consider one good and everything else.

EXAM

If one good is food and the other good is everything else, based on this relationship it tells you what happens when the price of food goes up.

Conclusion

We understand utility functions, how to maximize utility given a budget constraint, from that maximization process we can find demand functions, those demand functions can give us interesting analysis of goods, which allows us to solve elasticity questions. This allows us to solve interesting things such as if the price goes up what happens to how much I consume in dollar terms, and what impact does it have on the other goods that I consume.