

EXAMPLES OF CLASS 8 TOPICS

Cobb – Douglas Production Function:

The firm is going to produce as a function of two inputs, usually capital and labor.

The Cobb-Douglas production function has the following form:

$$q = f(k, l) = Ak^{\alpha}l^{\beta} \quad \text{where } A, \alpha, \beta > 0$$

It is a simple function but quite useful for practical purposes. Now, we want to analysis this function a bit, get an idea what this production function looks like and what some of the solutions for it are.

First is Returns to Scale (did this last week). Remember, returns to scale, if I increase my inputs by some portion what happens to the output is dependent on the shape of the production function. We will test it in this particular case. If I scale up my factors by some proportion, called t , so I grow my factors some value t , then in this particular functional form we have the following...

$$\begin{aligned} f(tk, tl) &= A(tk)^{\alpha}(tl)^{\beta} = (t)^{\alpha+\beta}Ak^{\alpha}l^{\beta} \\ &= t^{\alpha+\beta}f(k, l) \end{aligned}$$

So we see here that scaling each input by t results in our being able to factor t out, although raised by some power. The question is do we have increasing returns to scale or decreasing returns to scale? This depends on the value of $\alpha + \beta$. We know that they are positive, we defined them that way.

So we are going to have increasing returns to scale if $\alpha + \beta > 1$.

INCREASING

Why? Well if I increase everything, say double it all, my production will be more than doubled if $\alpha + \beta > 1$. So we find increasing returns to scale if $\alpha + \beta > 1$.

CONSTANT

A constant return to scale would simply be t times the production function. That will only happen if $\alpha + \beta = 1$. If I double the inputs I get double the output value.

DECREASING

We have decreasing returns to scale if $\alpha + \beta < 1$.

So depending on the constants in the original function we can have increasing, constant, or decreasing returns to scale.

We now wish to calculate the cost function. Simple idea, I know that if I want a fixed level of output, if I need to produce 10,000 units, there is going to be some optimal mix of labor and capital which will give me the minimum cost to produce those units. My cost function takes those different combinations of capital and labor, calculates the cost in each case, and therefore tells me, given any level of production, how much cost is it going to take me to produce that many units.

It's a two step process.

1. Tell me how many units we need
2. I tell you how much cost we will incur.

How do I know how much cost we will incur? Because I'm going to solve this to find the best combination of capital and labor that minimizes cost. That is the cost that I'm interested in. With this type of function I can produce a fixed output given many different combinations of k and l . But there is likely to be only one combination that is going to give me the minimum cost of producing those units. That's what the cost function is going to tell me.

To actually solve this what I have to do is find the best mix of units in order to minimize cost. How do we do this?

1) Construct a Lagrangian

$$L = wl + vk + \lambda[q_0 - f(k, l)]$$

$w \equiv$ Labor Rate, price or cost per labor hour.

$v \equiv$ Cost of Capital, usually expressed as an hourly rate, usually talking about machinery.

There is an easy way of visualizing this production function is that I need to produce widgets and I have various ways of doing so. I can have a very automated process and little labor or a mostly hands on process with much labor and few machines (capital investment).

Someone in the factory has worked out what the cost of operating this machinery is. They calculated an hourly cost for operating this machinery. So when I look at this function here my inputs are actually labor hours and hours I need to use the equipment. For example, imagine having labor and renting equipment. K is how many hours I've rented the equipment. V is the cost of capital, usually also an hourly rate.

q_0 , that's the fixed number of units I'm interested in. I'm trying to find the best mix of labor and capital in order to produce those units. How do I produce them? Of course I do it with a production function. This construction is the equivalent of the budget constraint in the consumer problem. I need to produce q_0 units, how am I going to do that? I will use k units of capital and l units of labor to do so. q_0 is equal to $f(k, l)$ when I am using the correct mix.

[resume class 9 tape 1 13:33]

NOTATION

$$F_k = \frac{\delta f(k,l)}{\delta k}, \quad F_l = \frac{\delta f(k,l)}{\delta l}$$

$w \equiv$ Labor Rate, price or cost per labor hour.

$v \equiv$ Cost of Capital, usually expressed as an hourly rate, usually talking about machinery.

Now we solve the LaGrangian maximization function by taking the three partial derivatives and set the results equal to zero. The solution below is general, meaning that we have not assumed anything about the form of the function $f(k, l)$.

$$L = wl + vk + \lambda[q_0 - f(k, l)]$$

$$\frac{\delta L}{\delta k} = 0 = v - \lambda \frac{\delta f(k,l)}{\delta k} \Rightarrow$$

$$\frac{\delta L}{\delta l} = 0 = w - \lambda \frac{\delta f(k,l)}{\delta l} \Rightarrow$$

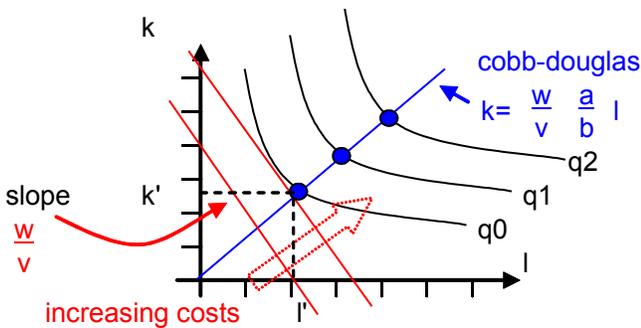
$$\frac{\delta L}{\delta \lambda} = 0 = q_0 - f(k, l) \Rightarrow q_0 = f(k, l)$$

$$v = \lambda F_k \Rightarrow \lambda = \frac{v}{F_k}$$

$$w = \lambda F_l \Rightarrow \lambda = \frac{w}{F_l}$$

eliminate λ to get $\frac{F_l}{F_k} = \frac{w}{v} = \frac{MP_l}{MP_k} = \text{RTS}$

This result for RTS takes us back to what we were talking about last class. Consider the graph:



The output q increases as the isoquants move northeast. To produce the units at q_0 optimally we use the labor and capital at l' and k' . The slope of the red lines is the Rate of Technical Substitution. When I employ different mixtures of labor and capital the slope of the red lines is the ratio of the prices of the two sources, the two factor inputs. The labor rate and the capital rate. This line shows different

combinations of labor and capital that cost the same amount. **So anywhere on this line is a mixture of capital and labor that cost the same in dollars.** Therefore as I go away from the origin, I'm getting more and more costly mixes of capital and labor. Therefore, the best mix of capital and labor to generate this number of units is the amount that just touches the isoquant. Don't want to go too far because that would increase the costs. Want to produce the amount q but do so at the cheapest cost. To find the cheapest cost I keep increasing my costs until I just touch the amount of production that I need (isoquant).

So the optimal point we are looking for is the point at which the point of the cost line is equal to the slope (tangent) of the isoquant. This is how we derive $RTS = \frac{w}{v}$.

The isoquant curves are generated by the equation $q = f(k, l)$. We'll see this below. To find the optimal mix, the best mixture of labor and capital to produce a given number of units at the cheapest cost set by the RTS equation (?).

The above is in very general terms, Now we will do an example using the Cobb-Douglas form of the units production value $q = f(k, l) = Ak^\alpha l^\beta$.

We need to calculate F_k and F_l :

$$F_k = \alpha Ak^{\alpha-1} l^\beta$$

$$F_l = \beta Ak^\alpha l^{\beta-1}$$

Now divide one by the other:

$$\frac{F_l}{F_k} = \frac{\beta Ak^\alpha l^{\beta-1}}{\alpha Ak^{\alpha-1} l^\beta} = \frac{\beta k}{\alpha l} = \text{RTS} = \frac{w}{v} = \frac{MP_l}{MP_k}$$

(this is specific to the Cobb-Douglas function)

So for whatever number of units we are asked to produce we can tell what the ratio of the factors need to be. We know this because we know the $\text{RTS} = w/v$. And therefore

$$\frac{\beta k}{\alpha l} = \frac{w}{v} = \text{RTS}$$

$$\frac{\beta k}{\alpha l} = \frac{w}{v} \quad \text{rearrange to solve for k} \quad k = \left(\frac{w}{v}\right) \left(\frac{\alpha}{\beta}\right) l$$

This expression for k is the equation of the **Expansion Path** of the Cobb-Douglas function. This is a straight line, given l we can specify k needed in order to be optimal. What we are saying is suppose we do this optimization problem for a number of different points (different outputs). (See graph above). This is called the expansion path because as the business grows, as it changes the number of units it produces, this is the mix of capital and labor that it uses each time.

Example:

Let $\alpha = \beta$, if the cost of labor is the same as the cost of capital then the expansion path has a 45 degree slope (equal amounts).

Now we want to know how many units to produce ...

Example:

$$w = 2 \text{ (capital cost)} \quad v = 5 \text{ (labor cost)} \quad A = 1 \quad \alpha = \frac{1}{2} \quad \beta = \frac{1}{4}$$

We wish to manufacture 1000 widgets given the parameters above and assuming my production function has the form $Ak^\alpha l^\beta$.

We can produce this number of units many different ways, can have many combinations of capital and labor. But which mix gives us the lowest cost? Well we know from our C-D solution if told how many of l to use we can specify k which is optimal.

$$\text{For optimal production } k = \left(\frac{w}{v}\right)\left(\frac{\alpha}{\beta}\right)l = \left(\frac{2}{5}\right)\left(\frac{1/2}{1/4}\right)l = \frac{4l}{5} \text{ (this is the expansion path eq).}$$

If I want to produce 1000 widgets how much k and l do I need? Plug in and solve...

$$1000 = Ak^\alpha l^\beta = \sqrt{k} \sqrt[4]{l} = k^{1/2} l^{1/4} \quad \text{substitute } k = \frac{4}{5}l \quad \text{then}$$

$$1000 = \left(\frac{4}{5}\right)^{1/2} l^{1/2} l^{1/4} \quad \text{solve for } l^{3/4} = \frac{1000}{\left(\frac{4}{5}\right)^{1/2}} \quad \text{then}$$

$$l = \left[\frac{1000}{\left(\frac{4}{5}\right)^{1/2}}\right]^{4/3} = 8254.8$$

$$k = \frac{4}{5}(8254.8) = 6603.8 \quad \text{and}$$

$$\text{Total Cost} = wl + vk = (\$2 \text{ per hr})(8254.8 \text{ hrs}) + (\$5 \text{ per hr})(6603.8 \text{ hrs}) = \$49,528.91$$

Cobb – Douglas Function General Solutions

If we know we are dealing with a cobb-douglas function we can use these equations to generate the solutions.

$$\text{for } f(k, l) = Ak^\alpha l^\beta \quad \text{the Expansion Path} = k = \left(\frac{w}{v}\right)\left(\frac{\alpha}{\beta}\right)l$$

where w is the labor rate and v is the capital rate.

$$l = \left(\frac{q_0}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} w^{\frac{-\alpha}{\alpha+\beta}} v^{\frac{\alpha}{\alpha+\beta}}$$

$$k = \left(\frac{w}{v}\right)\left(\frac{\alpha}{\beta}\right)l \quad \text{it follows that} \quad \text{Total Cost} = wl + vk$$

on the exam we are likely to see the form $A\sqrt{k}l$ this is a

Cobb-Douglas form where $\alpha = \beta = \frac{1}{2}$, aka $\alpha + \beta = 1$

Key for
EXAM
RTS = w/v

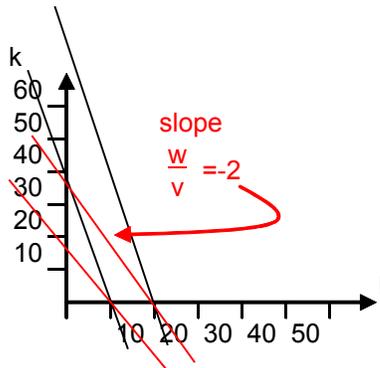
Example #3

Suppose I have a **linear** production function:

$$q = 4l + 1k \quad \text{where labor cost, } w = \$20/\text{hr} \text{ and capital cost, } v = \$10$$

(Important point: the q equation looks a lot like the total cost equation but it is not. The w and v go into the total cost equation to give Total Cost = 20l + 10k)

How should I manufacture 500 widgets? We do not have to go through the LaGrangian solution, we can see the solution on the graph!



Draw some isoquants, start with $q=40$. We can tell by the form of the q equation that the isoquants will be linear. Solve for a few isoquants:

$$q = 40 = 4l + 1k \rightarrow @ l=0, k=40 \text{ and } k=0, l=10$$

$$q = 80 = 4l + 1k \rightarrow @ l=0, k=80 \text{ and } k=0, l=20$$

We know the slope of the total cost function is

$$\frac{w}{v} = -2 = \text{RTS} \quad \frac{l}{k} = \frac{4}{1} \quad \text{a 4 to 1 ratio}$$

So we look at the isoquant q lines and think “how am I going to find the tangent of a linear line?” What we do in this case is look for the point where the lines, isoquant and w/v capital-labor, intersect.

If I want to manufacture 40 widgets, if I use all labor I have to use 10 hours of labor. If I use all capital I have to use 40 hours of capital. Now consider the cost of capital and labor. Labor cost me \$20 per hour and capital cost me \$10 per hour. If I use 10 hours of **labor my total cost is \$200**, if I use 40 hours of **capital it cost me \$400**. This is telling me it is cheaper to go the all labor route which happens to be the solution of the RTS intercept.

Capital and labor with this production function are perfect substitutes. It takes me $\frac{1}{4}$ of the labor hours compared to the capital hours. If labor cost me 20 and capital cost me 10, and I am trying to minimize my cost, the ratio of cost to labor from the q equation is 4 to 1, labor gets me to my q target 4 times faster than capital, with all labor I need 10 hours to get $q=40$ (costing me \$200) whereas with all capital I need 40 hours to get $q=40$ (costing me \$400).

So we see that all labor is the solution which minimizes cost, I use 10 hours of labor costing $\$20 \times 10 = \200 (as opposed to the cost of \$400 if I use all capital).

Keep in mind the red lines represent the combinations of labor and capital which give the same cost throughout the line. When can I manufacture the most for that cost? We have found that this occurs at the $k=0$ /all labor point because labor out produces capital 4 to 1.

In this case RTS is never equal to w/v . So the best solution with this cost is all labor and no capital. It takes $\frac{1}{4}$ hr to manufacture a widget with labor or 1 full hour with capital. One hour of capital cost \$10, a quarter of an hour of labor cost \$5. Therefore I use all labor and no capital!

The point is that we have to step back sometimes and look at what is really going on.

Last Example: It is cheaper to use all labor so I will need $500/4=125$ labor hours at a cost of \$2500 to produce 500 units.

Example #4

Pizzas are produced according to the following production function.

$$q = 20\sqrt{kl} \text{ if } w = \$20/hr \text{ and } v = \$10/hr$$

So we can see here that using zero capital of labor is not a possibility. Will have at least some of each. Find expressions for **Total Cost**, **Average Cost**, and **Marginal Cost**. We want the Long Run solution. The difference is that in the short run often things cannot be adjusted in time to meet the schedule. By Long Run we mean the firm has flexibility in deciding how equipment and how much labor they have, they are not constrained in any way. Start with Total Cost, how will we solve. Well we notice that the production function has the Cobb-Douglas form:

$$q = 20k^{1/2}l^{1/2} \text{ so it's a Cobb-Douglas with } A=20 \text{ and } \alpha = \beta = \frac{1}{2} \text{ so that } \alpha + \beta = 1$$

$$\text{Total Cost} = wl + vk$$

$$l = \left(\frac{q_0}{A}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{\beta}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta}} w^{-\frac{\alpha}{\alpha+\beta}} v^{\frac{\alpha}{\alpha+\beta}}$$

$$k = \left(\frac{w}{v}\right)\left(\frac{\alpha}{\beta}\right)l$$

So we can use our stock equations. Note we have not been asked for optimal solution.

Now we need to find values for l and k (we know w and v). We can use our standard formula to calculate solutions for l and k. We have not been asked to solve for a specific q so we will derive a **general q solution**. Plug in values to find...

$$l = \left(\frac{q_0}{20}\right)^{\frac{1}{1/2+1/2}} \left(\frac{1/2}{1/2}\right)^{\frac{1/2}{1/2+1/2}} 20^{-\frac{1/2}{1/2+1/2}} 10^{\frac{1/2}{1/2+1/2}} = \left(\frac{q_0}{20}\right)(1) \frac{\sqrt{10}}{\sqrt{20}} = \frac{q_0 \sqrt{10}}{\sqrt{20}\sqrt{20}\sqrt{20}}$$

$$k = \left(\frac{w}{v}\right)\left(\frac{\alpha}{\beta}\right)l = \left(\frac{20}{10}\right)\left(\frac{1/2}{1/2}\right)l = 2l = = q_0 \frac{2\sqrt{10}}{20\sqrt{20}} = q_0 \frac{1}{10} \sqrt{\frac{1}{2}}$$

$$\text{Total Cost} = wl + vk = 20l + 10k = \frac{1}{2}\sqrt{\frac{1}{2}}q_0 + \sqrt{\frac{1}{2}}q_0 = 2\sqrt{\frac{1}{2}}q_0$$

One unit is twice as costly as the other so you use 1/2 as much so that the COST from each is equal. In Cobb-Douglas the alpha and beta are kind of like the effectiveness of each unit. Since each is 1/2 in this case they are each as effective.

Here we call Total Cost the Cost Function because it relates the number of units to cost. Remember, we've taken an arbitrary number of units and found the most efficient way to produce them. This justifies the complexity. In this case the average and marginal costs are the same. **Marginal Costs is the rate of change of Total Cost, the rate at which costs are increasing. Average cost is giving us cost per unit on average. Marginal cost tells me how much additional I have to pay to produce an additional unit (one more unit).** True in this case because our cost curve is a

$$\text{Avg Cost} = \frac{\text{Total Cost}}{\# \text{ units}} = \frac{\text{Total Cost}}{q_0} = 2\sqrt{\frac{1}{2}}$$

$$\text{Marginal Cost} \equiv \frac{d}{dq}(\text{Total Cost}) = \frac{d}{dq}(2q\sqrt{\frac{1}{2}}) = 2\sqrt{\frac{1}{2}}$$

straight line. Avg Cost is the slope, Marginal Cost is the tangent.

Example #5

Manufacturing using Cobb-Douglas production function, $\alpha + \beta = 1$.

$$q = \frac{1}{10} k^{.2} l^{.8} \quad \text{where } \alpha = .2 \quad \text{and} \quad \beta = .8 \quad \text{and} \quad w = v = \$50/\text{hr}$$

How should the firm manufacture 10 units in most cost efficient way? Here we will use the long hand solution. **If we are looking for an optimal solution so we set RTS = w/v.** We will use this to find the functional form that we need.

$$\text{RTS} = \frac{w}{v} = \frac{\frac{\delta F_l}{\delta l}}{\frac{\delta F_k}{\delta k}}$$

$$\frac{\delta}{\delta l} \left(\frac{1}{10} k^{.2} l^{.8} \right) = \frac{.8}{10} k^{.2} l^{.8-1} = \frac{.8}{10} \left(\frac{k}{l} \right)^2$$

$$\frac{\delta}{\delta k} \left(\frac{1}{10} k^{.2} l^{.8} \right) = \frac{.2}{10} k^{.2-1} l^{.8} = \frac{.2}{10} \left(\frac{l}{k} \right)^8$$

RTS is equal to the ratio of these two solutions:

$$\text{RTS} = \frac{.8}{.2} \left(\frac{k}{l} \right)^2 \left(\frac{l}{k} \right)^8 = \frac{.8}{.2} \left(\frac{k}{l} \right) = 4 \left(\frac{k}{l} \right) = \frac{w}{v} = \frac{50}{50} = 1$$

$4 \left(\frac{k}{l} \right) = 1$ This says we use 4 units of capital for every unit of labor.

We solve for k to find the Expansion Path:

$$\therefore 1 = 4k \Rightarrow \text{EXPANSION LINE (PATH)} = k = \frac{1}{4} l$$

So to manufacture 10 bats we set $q=10$

$q = f(k, l) = \text{isoquant}$

isoquant = $.1k^{.2}l^{.8} = 10 = q$ use $l = 4k$ to solve for k

isoquant = $.1k^{.2}(4k)^{.8} = 10$

isoquant = $.1k(4)^{.8} = 10$

solve for $k = \frac{10}{(.1)(4^{.8})} = 32.9$ which leads to $l = 4k = 131.95$

Total Cost = $33 \cdot 50 + 132 \cdot 50 = 8250$

Solve using formula solution

$$1 = \left(\frac{10}{.1} \right)^{\frac{1}{.2+.8}} \left(\frac{.8}{.2} \right)^{-2} 50^{-.2} 50^{.2} = 100(4)^2 = 131.95, \quad k = \left(\frac{.2}{.8} \right) \left(\frac{50}{50} \right) l = \frac{132}{4} = 33 \text{ hours}$$

Example #5

Short Term versus Long Term. Firm manufacturing baseball bats.

Production Function: $q = 2\sqrt{kl}$ Cobb-Douglas with $\alpha = \beta = .5$, $w = \$4/\text{hr}$, $v = \$1/\text{hr}$
 In the short run $k = 100$ hours. If this is our production function for the next month, I've already rented 100 hours worth of equipment and I am constrained by this. When I solve my optimization problem I may want to use more or less than 100 hours but we will not be able to. It's a sunk cost, must use it all.

Since alpha and beta are each equal to $\frac{1}{2}$ there is equal value to using labor and capital. But here capital is much cheaper than labor I want to if possible use 4 times as much capital hour. But we cannot because of the constraint.

Start by calculating the Short Run Total Cost Curve (STC) This is manufacturing cost based on quantity demanded with the rental hours constraint. This is actually very simple. This is **because k is fixed in the short run**:

$$\text{simplified production function: } q = 2\sqrt{100l} = 20\sqrt{l}$$

So here we've related labor to quantity.

IF HE TELLS US THAT ONE OF THE FACTORS IS FIXED (CONSTRAINT) THE QUESTION BECOMES A TRIVIAL MATTER.

Short Run Total Cost = $STC = wl + vk = 4l + 100(1) = 4l + 100$

Still need to find the cost as a function of the number of units produced. Use the result $q = 20\sqrt{l}$ and solve for l:

$$l = \frac{q^2}{400} \text{ so } STC = 4l + 100 = \frac{q^2}{100} + 100 = \frac{1}{q} \left(\frac{q^2}{100} + 100 \right) = \frac{q}{100} + \frac{100}{q}$$

$$\text{Short Term Marginal Cost} = \frac{\delta}{\delta q} (STC) = \frac{\delta}{\delta q} \left(\frac{q^2}{100} + 100 \right) = \frac{q}{50}$$

Marginal is telling me if I manufacture 100 baseball bats then the next one cost an additional $101/50 = \$2.02$. Higher quantities become more inefficient because of k constraint, we are having to use more and more inefficient labor.

Important Point: If I can receive a certain price for baseball bats I'm only going to manufacture up to the point where the marginal cost is equal to the price. If it cost me more to manufacture that next baseball bat then I can sell it for I stop making them. If I'm a firm in the market place selling my goods and I calculate my marginal costs, I'm only going to sell (manufacture) units as long as the price is greater than the marginal cost. If I can't make it for what I can sell it for I don't make it.

Marginal cost tells me how much more it cost to build 1 additional unit.

SHORT TERM MEANS THERE IS A CONSTRAINT ON K OR L

Example #6

Ernie's lawn mowing business has the following cost function

$$\text{Cost Function} = C = .1q^2 + 10q + 50 \quad \text{where } q = \text{acres cut per day}$$

Ernie is a PRICE TAKER. He cannot effect the price of the service in the marketplace. He takes the price the market gives.

Price per acre is \$20. How many acres should Ernie cut per day? We know cost and profit per acre. **How many acres should Ernie cut per day?** Ernie wants to cut as many acres as he can to **maximize his profits**. How do we structure this problem?

define the function $\pi(q)$ as **daily profit**

$$\pi(q) = \text{Daily Revenue} - \text{Daily Cost}$$

$$\pi(q) = R(q) - C(q) \quad \text{where } R = \text{daily revenue and } C = \text{cost, a function of acres cut.}$$

Profit is a function of how many acres he cuts per day.

To maximize profits set the derivative of profits wrt the variable q to 0. We also need the second derivative to be < 0. q = optimal number of acres.

to maximize profit set $\frac{\delta\pi}{\delta q} = 0$ also need $\frac{\delta^2\pi}{\delta q} < 0$ where q = optimal number of acres

$$\frac{\delta}{\delta q} \pi(q) = \frac{\delta}{\delta q} R(q) - \frac{\delta}{\delta q} C(q) = 0 \quad \text{so we have} \quad \frac{\delta R}{\delta q} = \frac{\delta C}{\delta q}$$

Saying that the rate in change of revenues will be the same as the rate in change of cost. The thing on the right hand side is **Marginal Cost**. We will stop cutting lawns when it starts to cost me more to cut an additional area of lawn then I am going to receive for doing so.

Since Ernie is a price taker, revenue is equal to **p times q** where p is equal to the sale price which is \$20 per acre. [Note that if someone is not a price taker, if someone sees the demand curve that is downward sloping, then p could also be a function of q. The more I sell the less price I get since I'm flooding the market. Therefore p could also be a function of q.]

$$R = pq = 20q \quad \therefore \frac{\delta R}{\delta q} = 20 = \text{price!}$$

Derivative of R wrt q tells me how my revenues change as I cut additional acres. In this case we get another \$20.

$$C = .1q^2 + 10q + 50 \quad \therefore \frac{dC}{dq} = .2q + 10$$

$$\frac{dR}{dq} = \frac{dC}{dq} \Rightarrow 20 = .2q + 10, \quad \text{solve for } q = \frac{10}{.2} = 50 \quad \text{is the optimal number of acres to cut.}$$

Now calculate the second derivative to make sure it is a maximum:

$$\frac{d^2\pi}{dq^2} = \frac{d^2R}{dq^2} - \frac{d^2C}{dq^2} = 0 - .2 = -.2 < 0 \text{ GOOD!}$$

$$\text{Profit} = R - C = 20q - (.1q^2 + 10q + 50) = 20q - .1q^2 - 10q - 50$$

$$\text{Profit} = 20 \times 50 - .1 \times 50^2 - 10 \times 50 - 50 = \$200$$

Next Class, questions, new and ours. Consumer behavior as well similar to this format.