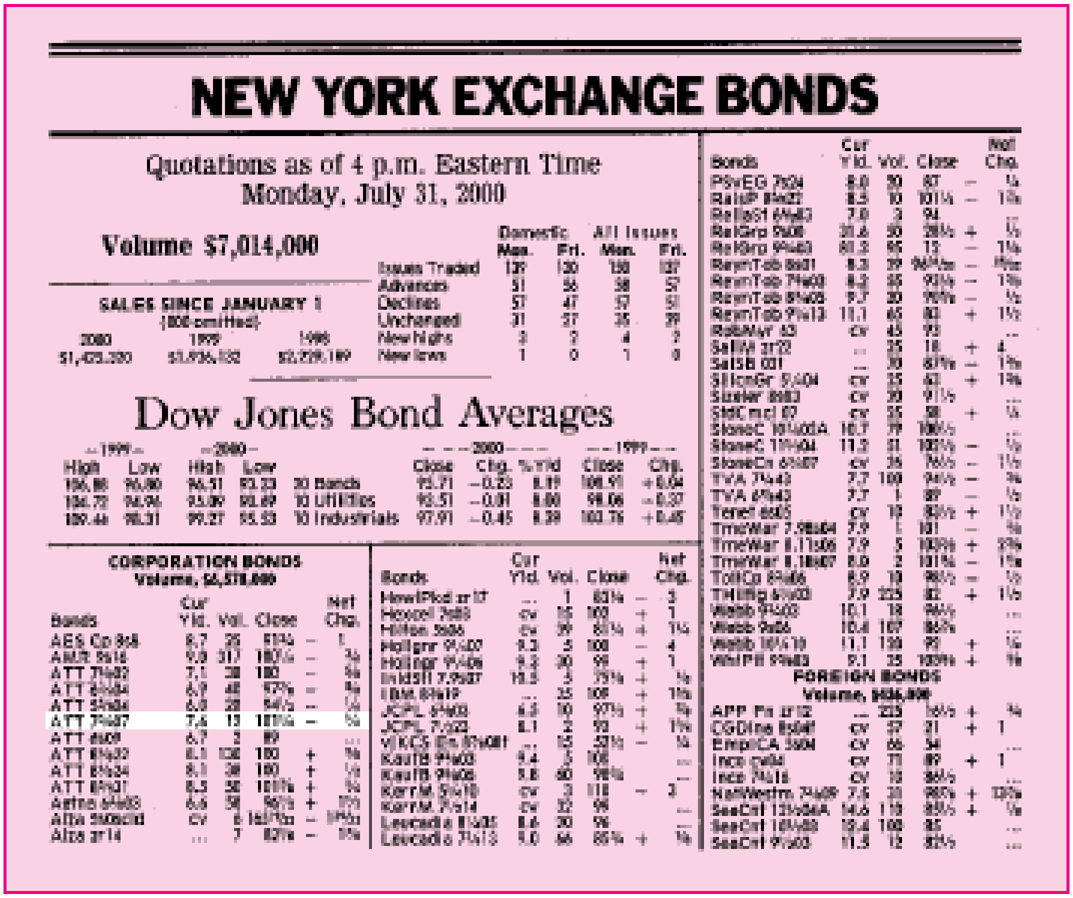
Valuation of Bonds



NEW YORK EXCHANGE BONDS

Quotations as of 4 p.m. Eastern Time
Monday, July 31, 2000

Volume $7,014,000

	Domestic Mon.	Domestic Fri.	All Issues Mon.	All Issues Fri.
Issues Traded	139	130	158	157
Advances	51	56	58	57
Declines	57	47	57	51
Unchanged	31	27	35	39
New highs	3	2	4	2
New lows	1	0	1	0

SALES SINCE JANUARY 1
(000 omitted)

2000	1999	1998
$1,425,200	$1,936,132	$2,229,189

Dow Jones Bond Averages

1999 High	1999 Low	2000 High	2000 Low		2000 Close	2000 Chg.	2000 %Yld	1999 Close	1999 Chg.
106.88	96.80	96.51	93.23	20 Bonds	95.71	−0.23	8.19	106.91	+0.04
104.72	94.96	95.09	92.69	10 Utilities	93.51	−0.01	8.00	98.06	−0.37
109.46	98.31	99.27	93.53	10 Industrials	97.91	−0.45	8.39	103.76	+0.45

CORPORATION BONDS
Volume, $6,578,000

FOREIGN BONDS

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Overview

- ❑ Features of securities and markets
- ❑ Valuation
- ❑ Yield to Maturity
- ❑ Risk
- ❑ Duration

What we are after here is the calculation for DVO1

DVO1 is the dollar value change in the price of a security given a one basis point change in the interest rate. This is a measure of price sensitivity, scale factor. Used throughout the course, hedging, futures contracts, options…

What is a Bond?

- A long-term debt instrument in which a borrower agrees to make payments of principal and interest, on specific dates, to the holders of the bond

Bond Markets

- Primarily traded in the over-the-counter (OTC) market (electronic network of dealers and brokers)
- Most bonds are owned by and traded among large financial institutions (banks, insurance companies)
- Full information on bond trades in the OTC market is not published, but a representative group of bonds is listed and traded on the bond division of the NYSE

There is no centralized place although there are some pay-for-use places on the web where they accumulate this data across the market, but not centralized.

Some bonds traded on the NYSE.

You can do some incredible things with bond info such as figure out the market risk of a bond, use a bond in a CAPM sense to measure its beta. Can use bond trading on the NYSE to try and estimate bond liquidity premiums. If a bond does not trade very much then investors are going to need a higher return to hold that bond. So you use the bond activity to estimate what investors need in terms of return to hold less trading bonds.

Key Features of a Bond

- PAR VALUE – face amount of the bond, which is paid at maturity (assume $1,000) (aka Maturity Value)
- COUPON INTEREST RATE – stated interest rate (generally fixed) paid by the issuer. Generally semi-annual.
- Multiply by par value to get dollar payment of interest.
- MATURITY DATE – years until the bond must be repaid
- ISSUE DATE – when the bond was issued
- YIELD TO MATURITY - rate of return earned on a bond held until maturity (aka "promised yield")

We say the YTM definition is "almost true" because it can be affected by reinvestment risk whereas YTM assumes you can reinvest the COUPON at the same rate the bond returns!

Effect of a Call Provision

- Allows issuer to refund the bond issue if rates decline
  - Helps the issuer, but hurts the investor
- Borrowers are willing to pay more, and lenders require more, for callable bonds
- Most bonds have a deferred call and a declining call premium

The US Treasury no longer issues bonds with call provisions so this is purely a corporate topic. This is similar to a homeowner refinancing their mortgage when rates go down. I pay back the original debt and barrow money at the original rate. This is what a call provision does for a company, they sell a bond when the market is in a certain place. then is market interest rates decline 9for a similarly positioned bond) the CALL allows them to repurchase that bond and then reissue debt at a lower rate.

If I have two bonds identical in every way except that one is callable and the other is not, then the YTM on the callable bond has to be at least the same as the original bond or greater. Investors require a higher return to own that bond due to the risk that it may be called.

Most callable bonds have a "deferred call" meaning there is a period in the beginning of the bonds life when it cannot be called. For example, if the term of the bond is 10 years it may have a deferred call of the first 5 years.

When the company that issued the bond calls it they usually have to pay a premium to the face value to buy it back. They do not usually have the opportunity to buy it back at face value. However, it is often the case that the price they have to pay to call it declines the closer the bond gets to maturity.

What is a Sinking Fund?

- Provision to pay off a loan over its life rather than all at maturity
- Similar to amortization on a term loan
- Reduces risk to investor, shortens average maturity / duration
- But not good for investors if rates decline after issuance

The bond will have a provision which says the company must maintain cash on its books (or must maintain a revolving line of credit) and they must repurchase parts of the bond over the life of the bond. They are continuously paying the principle back rather than at the end.

Also effects DV01 because DVO1 is linear in duration. Reduces interest rate risk to investor.

Valuation of Financial Securities

- First Principles:

> Value of Financial Securities = PV of Exp Future Cash Flows
> (this applies to any financial instrument)

- To value financial securities we need to:
  - Estimate future cash flows:
    - Size (how much) and
    - Timing (when)
  - Discount future cash flows at an appropriate rate:
    - The rate should be appropriate to the risk presented by the security.

For a bond this is the remaining coupon payments and the face value. But there may also be an expected value adjustment because there is concern the company is going under. The Expected Value is the estimated cash flows.

Discount rate is the required rate of return, market return. This RoR is the Yield to Maturity for a bond which is a market driven rate. Will be based on risk, greater risk means higher discount rate required.
Yield to Maturity on the bond is NOT the coupon rate!

Bond Valuation Process

- The appropriate bond price reflects the present value of the cash flows generated by the bond (i.e., interest payments and repayment of principal):

Coupon payments are not absolutely fixed (guaranteed) unless it is a government bond. If the issuer goes under you will not receive your coupon.

So we calculate the value of the bond by summing the value of the future cash flows discounted by required rate of return k (aka r).

$$PV = \frac{C}{(1+k)^1} + \frac{C}{(1+k)^2} + \cdots \frac{C+PAR}{(1+k)^n}$$

On ***BONDS*** the ***k*** is the ***YIELD TO MATURITY***.

Bond Prices and Yields

- Prices and Yields (required rates of return) have an *inverse relationship*
- When ***yields get very high*** the ***bond value will be very low***

$$as\ \mathrm{k} \to \infty, \qquad PV \to 0$$

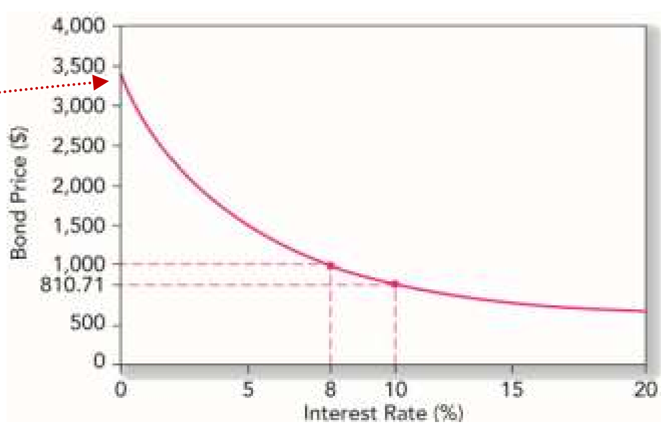
- When ***yields approach zero***, the bond value approaches the ***sum of the cash flows***

$$as\ \mathrm{k} \to 0, \qquad PV = \frac{C}{(1)^1} + \frac{C}{(1)^2} + \cdots \frac{C+PAR}{(1)^n}$$

For example, say a bond is selling for \$0.15 on the dollar. It is very low, people think the coupon payments are going to end some time in the future. k, the RoR, becomes very high and as a result the value of the bond decreases significantly.

THERE IS AN INVERSE RELATIONSHIP BETWEEN YIELDS AND PRICE.

The Inverse Relationship Between Bond Prices and Yields



For the bond characteristics listed above, if the interest rate is 0 then the bond price is the sum of the cash flows. As the yield declines the value of the bond goes up (and visa-versa).

When Coupon Payments equal the Yield to Maturity then Price equals Par Value.

Bond Valuation Process

- Impact of the timing of payments on bond valuation
  - ∗ Funds received sooner can be reinvested to earn additional returns
  - ∗ A dollar to be received soon has a higher present value than one to be received later

Most bonds pay a semi-annual coupon. If I compare two identical bonds where the only difference is that bond #1 pays annual coupons and bond #2 pays semi-annual coupons, bond 2 is more valuable. I receive the same total C every year but the semiannual coupon bond gets me my money sooner which I can reinvest so #2 is more valuable. This means that the price of the semi-annual bond MUST be HIGHER than the price of the annual coupon bond.

$Y_{SA} = \frac{Y_A}{2}$ Technically, $Y_{SA} = (1+Y_A)^{\frac{1}{2}} - 1$ but we will not be doing this math (?).

If prof gives us a semi-annual bond problem with an annual yield, just divide the annual by 2, don't have to do the technical compounding calc above. Assume nominal value, just double the semiannual.

VALUATION OF BONDS with **SEMIANNUAL PAYMENTS**

1. First, divide the annual coupon by two
2. Second, divide the annual discount rate (k/ytm) by two
3. Third, double the number of years

$$PV = \frac{\frac{C}{2}}{(1+\frac{k}{2})^1} + \frac{\frac{C}{2}}{(1+\frac{k}{2})^2} + \cdots \frac{\frac{C}{2} + PAR}{(1+\frac{k}{2})^{2n}}$$

Assume semi-annual if not told otherwise.

EXAMPLE: Price: 10-yr, 8% Coupon, Face = $1,000

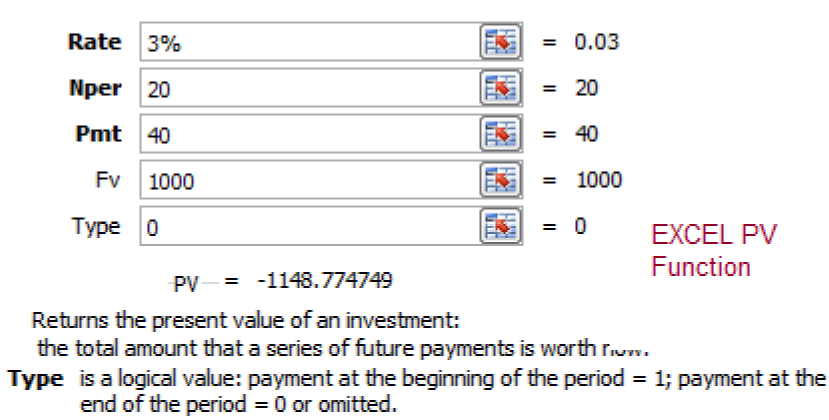
$$P = 40\sum_{t=1}^{20} \frac{1}{(1.03)^t} + \frac{1000}{(1.03)^{20}} = \$1148.77$$

(20 six month periods)

[redacted]

(semi-annual)

r (given) is the YTM we use to calculate the price of the bond



EXCEL PV Function

Yield to Maturity

- Interest rate that makes the present value of the bond's payments equal to its price
- Solve the bond formula for Yield to Maturity = r

r is the discount rate (YTM) which sets the bond value equal to the present value. Yield to Maturity: the discount rate on a series of bond payments which makes the bond value equal to the present value. Solve the below eq for r.

$$P=\sum_{t=1}^{T}\frac{C_t}{(1+r)^t}+\frac{Par\,Value}{(1+r)^T}$$

Cannot solve algebraically, must use Excel or Financial Calc.

If asked for annual YTM: Annual $_{\text{YTM}}$ = 2 * r

EXAM

Yield to Maturity Example

[redacted]

$$950=\sum_{t=1}^{20}\frac{35}{(1+r)^t}+\frac{1000}{(1+r)^{20}}$$

$$\boxed{\mathbf{r = 3.8635\%}}$$

Yield to Maturity

Excel function returns Annual, divide by 2 for semi-annual

Data	Description
1-Jan-00	Settlement date
1-Jan-10	Maturity date
7.00%	Percent coupon
95	Price (per $100)
$100	Redemption value
2	Frequency is semiannual
0	30/360 basis (see above)
7.73%	= Annual YTM
3.86%	**= Semi-Annual YTM**

The $950 price comes from the market, the last or most recent price at which this or a similar security was traded. 7% coupon means $35 semi-annual coupon. Face value is $1000 with 10 years to maturity , semi-annual coupon payments means 20 periods. If asked for annual YTM: Annual $_{\text{YTM}}$ = 2 * r

Relationship between

Coupon Rate, Required Return, and Price

- If the coupon rate of a bond is below the investor's required rate of return, the Present Value of the bond should be:

Below PAR Value → Selling at a discount, Coupon r below YTM.

When Coupon Rate < YTM ⟺ PRICE < PAR Value

(*DISCOUNT BOND*)

Coupon rate is below YTM, investors expect a 10% return but the coupon is 8%, How are they going to get that additional 2%? Answer, the bond sells for below its PAR value and over the life the investor receives a capital gain.

- If the coupon rate equals the required rate of return, the price of the bond should be equal to par value

When Coupon Rate = YTM ⟺ PRICE = PAR Value

If this does happen it would be only briefly. Suppose coupon rate is 10% and the required return is 10%, the investor gets the return the market demands.

- If the coupon rate of a bond is above the required rate of return, the price of the bond should be

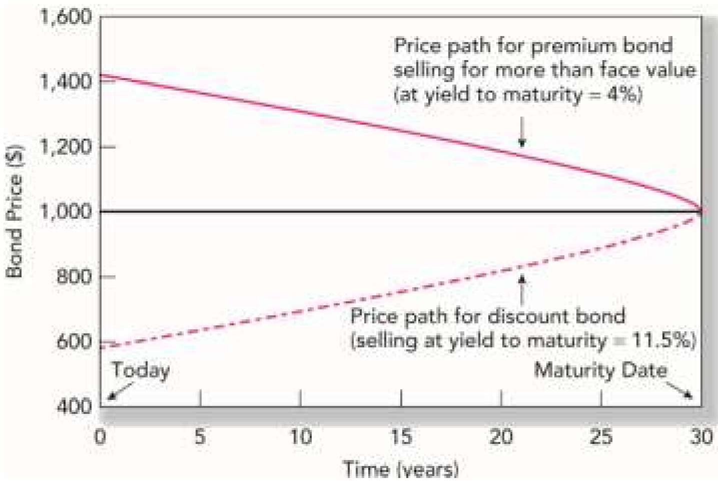
Above PAR Value → Selling at a premium, Coupon r below YTM.

When Coupon Rate > YTM ⟺ PRICE > PAR Value

(*PREMIUM BOND*)

If the coupon is above, the price must be above. Why? If coupon is higher than required return the investor has to lose capital to make up for that difference.

Prices over Time of 30-Year Maturity, 6.5% Coupon Bonds



This diagram shows the market price of a bond over its life. The values we see here do not include accrued interest. Assuming the YTM does not change over the life of the bond (ideal), a bond which sells at a premium is going to decline in value until at the end of its life its value is equal to PAR Value.

Likewise, if the bond sells at a discount it will go up in price until at the end of its life its value is equal to its PAR Value.

In the real world the market yield will change constantly and the paths in the above diagram will be more of a random path. But it is certain that at the end the bond will be worth its PAR Value. Also, if we include **accrued interest** the paths would have a saw-tooth shape because every 6 months the value would go up due to the interest earned over the life of the bond. So this diagram is called a "**price pre-accrued interest**" diagram. (price which is published in the news paper (?).

Definitions

$$\text{Current yield (CY)} = \frac{\text{Annual coupon payment}}{\text{Current price}}$$

$$\text{Capital gains yield (CGY)} = \frac{\text{Change in price}}{\text{Beginning price}}$$

$$\text{Expected total return} = \text{YTM} = \begin{pmatrix}\text{Expected}\\ \text{CY}\end{pmatrix} + \begin{pmatrix}\text{Expected}\\ \text{CGY}\end{pmatrix}$$

Yield to Maturity

Excel function returns Annual, divide by 2 for semi-annual

Data	Description
1-Jan-00	Settlement date
1-Jan-10	Maturity date
9.00%	Coupon Annual Percent
88.7	Price (per $100)
$100	Redemption value
1	Frequency is Annual
0	30/360 basis (see above)
10.91%	= Annual YTM
5.46%	= Semi-Annual YTM

CURRENT YIELD: Measures only the annual interest income the bondholder receives as a percentage of the price paid for the bond. It ignores the PAR payment at maturity date even if investor paid less than Par for the bond. Prospective price appreciation or depreciation does not enter the computation of the current yield. If the bond is selling for its PAR value the current yield will equal the coupon value.

CAPITAL GAINS YIELD: The price appreciation component of a security's (such as a common stock) total return. For stock holdings, the capital gains yield will be the change in price divided by the original (purchase) price. Change in price from the beginning of a period to the end of the period divided by the beginning price.

$$\text{Capital Gains Yield} = \frac{P_1 - P_0}{P_0}$$

EXPECTED TOTAL RETURN: The sum of dividend yield (dividend/price) plus the expected dividend growth rate, g. Expected Total Return is equal to the Yield to Maturity. I can only earn a return by investing in a bond one of two ways. Either from receiving a coupon check or because the price of the bond goes up (or down). Therefore the expected total return has to be the sum of the expected current yield plus the expected capital gains.

$$E(r) = \frac{D_1}{P_0} + g$$

In the real world prices move all over the place and you never get what you expect.

EXAM

EXAMPLE: Find the current yield and the capital gains yield for a 10-year, 9% annual coupon bond that sells for $887, and has a face value of $1,000

Current Yield = $90 / $887 = 0.1015 = 10.15%

Capital Gains Yield = YTM – CY = 10.91% - 10.15% = 0.76%

Expected Total Return=YTM=CY + CGY=10.59% + .76% = 11.35%

Could also find the expected price one year from now and divide the change in price by the beginning price, which gives the same answer

Messy if you calculate for a semi-annual bond. (Should look up for exam).

Interest Rate Risk

- Suppose a company has an obligation falling **due in 4 months** for $10 million
  - The company wishes to ensure it has sufficient funds at that date so decides to invest in T-bills today
  - Suppose the only available T-bill **matures in 6 months**
    - It currently sells for 98% of its face value
  - Assuming market rates stay constant, how much should the company invest? (in order to make the $10 million payment in 4 months)

Company wants to invest money today so that they will have $10 million 4 months from now. Company will have to invest in the 6 month T-bill and sell it on the open market in 4 months. The T-Bill is currently selling at a discount of 2% below its face value.

The bond is selling at $98 per $100 face value. Therefore, over the six month life the yeild to maturity that I am going to earn will be 2.04%.

The T-bill will pay R = (100/98) – 1 = 2.04% over its six month life. This return is guaranteed, no risk. But the company cannot hold the T-Bill for 6 months…

The return over 4 months will be $R = \left(\frac{100}{98}\right)^{4/6} - 1 = 1.36\%$, note R is the 6 month rate. Assuming rates do not change

The 1.36% is only an expected return, market rates can change. But this is the best bet of what the return will be over the 4 month period based on the6 month rate. Now we can calculate how much we need to invest.

In order to have $10 million in 4 months the company will invest …

$10 million / 1.0136 = $9,866,218

What is the face value of the T-Bills I will purchase if they are selling at 98% of face value?

$$\$9,866,218 \times \frac{100}{98} = \$10,067,570$$

Everything goes well until the day comes when the company needs the money. They find that market rates have increased to the point where the company only receives $9980,174…

Company is short by: $10 million – 9,980,174 = $19,826 …

The actual return to the company indicates the interest rates have gone up by approximately 25 basis points.

THIS IS AN EXAMPLE OF INTEREST RATE RISK !!!

I've bought an asset. the price of the asset falls because of a change in market rates, so that when I sell the asset I receive less then I was projecting.

Interest rate risk can also work in your favor. If the rates had gone down the asset could have been sold for a profit, over and above the projection. Risk is variability, I can benefit from it or I can lose from it.

When you are long in bonds, you can be hurt by interest rate risk when rates go up.

Reinvestment Risk (same situation as example above)

- Suppose the company still has an obligation falling due in 4 months for \$10 million
  - Once again, the company wishes to ensure it has sufficient funds at that date so decides to invest in T-bills today
  - Suppose the only available T-bills matures in 2 months
    - It currently sells for 99% of its face value
- Suppose at the end of the first two-month period there will be another 2-month t-bill to invest in (reinvest proceeds of first inv)
- Assuming market rates stay constant, how much should the company invest?

Return over **2** months will be: $\left(\frac{100}{99}\right) - 1 = 1.01\%$ Assuming rates do not change…

Over the **4** months, the return will be $\left(\frac{100}{99}\right)^2 - 1 = 2.03\%$. (compounding the 1.01%)

So to have \$10 million in 4 months, invest $\frac{\$10,000,000}{1.0203} = \$9,801,000$

The face value of the first investment is \$ 9,801,000 x 1.0101 = **\$9,900,000**

Suppose the yield decreases during the first period so that at the time to reinvest, rates have decreased by 25 basis points. (basis point = 1%/100 = .01/100 = 0.0001)

At reinvestment, the new rate will be 1.01% - 0.25% = 0.76% At reinvestment we have **\$9,900,000** to invest. This will yield **\$9,900,000** x 1.0076 = \$9,975,250

So we are short by \$10 million – 9,975,250 = \$24,750 due to the decrease in interest rates by the 25 basis points.

When you are long in bonds, you can be hurt by reinvestment ate risk when rates go down.

At the end of the semester we will look at interest rate futures to see how we can reduce interest rate risk and reinvestment risk. Perhaps go short in interest rate, bonds, Eurodollars

EXAMPLE:

Hidden Investment Risk. Suppose I have $10 Million to invest. I wish to buy a 2 year T-Note, the most recent coupon was paid yesterday (meaning we do not have to worry about calculating accrued interest).

Coupon Rate: 4.5% Annual, Price 101:00, PAR Value = $1000, Price = $1010 (it is trading at a slight premium).

Q1. What is the yield to maturity?

Price = $1010 Coupon = $\$1000 \times \frac{4.5\%}{2} = \$22.50$

Par Value = FV = $1000

n = 2 years @ 2 payments each = 4

YTM (semi-annual) = 1.987%

YTM (nominal Annual) = 2 * 1.987% = 3.974%

(nominal means true YTM doubled, you never actually earn the 3.974% on an annual basis, it's a nominal value)

Yield to Maturity

Excel function returns Annual, divide by 2 for semi-annual

Data	Description
1-Jan-00	Settlement date
1-Jan-02	Maturity date
4.50%	Coupon Annual Percent
101	Price (per $100)
$100	Redemption value
2	Frequency is SemiAnnual
0	30/360 basis (see above)
3.975%	= Annual YTM
1.987%	= Semi-Annual YTM

Q2. What return am I expecting over the two year period?

I'm investing $10 Million, YTM is a market return. So what is the earning over 2 years? What percentage can we apply to the $10 Million to arrive at the answer?

True YTM for a 6 month period is 1.987%. Compound for 2 years 94 periods).

2 year YTM = $(1+1.987\%)^4 - 1 = 8.188\%$

Q3. How much capital do I expect to have at the end of the two year period?

Expected Earnings over the two years: $10,000,000*(1+8.188%)=**$10,818,800**.

Q4. How many bonds will I buy (at $1000 ea)? The bonds are trading at $1010 each: $\frac{\$10,000,000}{1010} = 9,900.99$ bonds to purchase.

(cont. next page)

Q5. Add up the cash flows from the bond investment.

We know from Q3 that we expect to have $10,818,800.

There are 4 coupons and 1 Par Value payment.

Year	Type	Cash	
0.5	C	$9901*22.50=	$222,773
1	C	$9901*22.50=	$222,773
1.5	C	$9901*22.50=	$222,773
2	C	$9901*22.50=	$222,773
2	Par	$9901*1000=	$9,901,000
		Total:	$10,792,090

So we are short of the $10.8 million we had expected. Why? The reason is that we must reinvest our interim cash flows for the remaining periods.

Yield to Maturity depends on maturity! The longer the time to mature the higher the return (generally).

Our goal would be to reinvest interim cash flows at the same rate we calculated in part 1. But there is no chance it would be the same rate. Interest rates are constantly moving. If they decline in the middle of your project and you have not planned there will be trouble! Reinvestment risk!

Zero Coupon bonds will guarantee a return, there is no reinvestment risk. Same is true for CDs.

The longer to maturity the greater the reinvestment risk.

Price Sensitivity and Maturity

- In general, the longer the term to maturity, the greater the sensitivity to interest rate changes
- Bonds with identical maturities will respond differently to interest rate changes when the coupons differ
  - This is more readily understood by recognizing that coupon bonds consist of a bundle of "zero-coupon" bonds
  - With higher coupons, more of the bond's value is generated by cash flows which take place sooner in time
  - Consequently, less sensitive to changes in market return (larger coupon rate giving you your money back sooner which lowers the duration)
- The longer maturity bonds experience greater price changes in response to any change in the discount rate
- **The range of prices is greater when the coupon is lower**
  - The 6% bond shows greater changes in price in response to a ±2% change than the 8% bond
  - The first bond is has greater interest rate risk

Price Sensitivity of a 6% Coupon Bond (n years)				
	Yield to Maturity			
n (yrs)	8%	6%	4%	Range
40	$802	$1,000	$1,273	$471
20	$864	$1,000	$1,163	$299
10	$919	$1,000	$1,089	$170
2	$981	$1,000	$1,019	$38

Price Sensitivity of a 8% Coupon Bond (n years)				
	Yield to Maturity			
n (yrs)	10%	8%	6%	Range
40	$828	$1,000	$1,231	$403
20	$875	$1,000	$1,149	$274
10	$923	$1,000	$1,085	$162
2	$981	$1,000	$1,019	$38

Interest rate risk, when interest rates change the value of the bond changes. Interest rates up, value of bond down – interest rates down, value of bond increases.

At this point we want to look at how much does the value of the bond change when interest rates change and what characteristics of the bond make the changes greater.

The longer the term to maturity the greater the sensitivity to interest rate changes. longer term to maturity, greater interest rate risk. Why? Longer term to maturity means your cash flows are way off in the future and as a result the compounding effect of interest has the greatest effect on the bond. If I have a bond that matures tomorrow I do not care how interest rates change, it will not effect the value of my bond. If my bond matures in 30 years changes in the interest rate will have a big impact. The longer the term to maturity the greater the impact to changes in interest rates.

Bonds with identical maturities will respond differently to interest rate changes based on the size of their coupon. If we think of a bond as being a sequence of zero-coupon bonds and then a big lump sum at the end. In general, the higher the coupon rate the more value is pulled forward (get your money sooner). Think of the present value calculation, calculate the present value of all the Cs then the lump sum. The bigger the Cs the more value is closer to today. It is actually the same effect as the maturity, it's just more disguised. The larger the coupon the less sensitivity there is to changes in interest rates.

Zero-Coupon bond has the lowest coupon you can get. Therefore, for a fixed maturity, the zero-coupon bond suffers the greatest interest rate risk (because you get no payments as compared to another bond with the same maturity which does make coupon payments). We will see that the zero-coupon bond, for any maturity, has the longest duration. For a zero-coupon bond, duration is equal to maturity. (see below).

Price Sensitivity of a 6% Coupon Bond (n years)				
	Yield to Maturity			
n (yrs)	8%	6%	4%	Range
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10	$919	$1,000	$1,089	$170
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Here we have different yield to maturities. Notice when the YTM is 6% each of the four bonds in the 6% column has a **Price** equal to **Par Value**. This is because **YTM is equal to the coupon rate.** Now we change YTM by ±200 basis points. What is the impact on the bond?

Price Sensitivity of a 8% Coupon Bond (n years)				
	Yield to Maturity			
n (yrs)	10%	8%	6%	Range
40	$828	$1,000	$1,231	$403
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10	$923	$1,000	$1,085	$162
2	$981	$1,000	$1,019	$38

The two year bond (shortest maturity) has a range of $38. The 40 year bond has a range of $471, this is because it has the longest time until maturity, it suffers the most change in value.

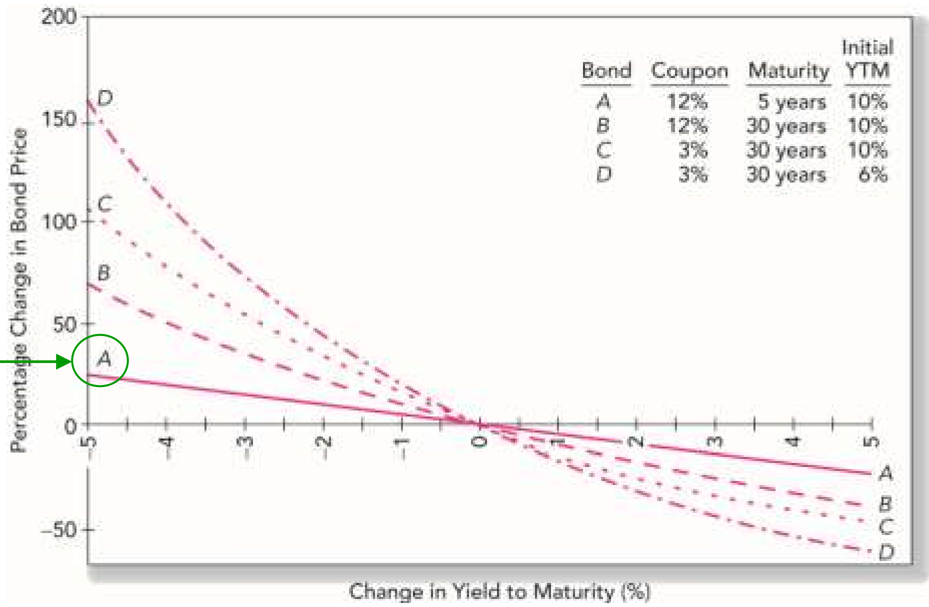
Now the 8% coupon bond, still have the ±200 basis point change. So this bond has a larger coupon rate, yet for the same maturity, each of the ranges is less (except for 2 year which is equal). Bottom line:

THE LARGER THE COUPON RATE THE LESS THE SWING (RANGE).

Long maturity bonds experience greater price changes (response to changes in discount rate / YTM). The range of prices is greater when the coupon is lower.

Change in Bond Price as a Function of Change in Yield to Maturity

A: lowest maturity, highest coupon.



Bond A has the lowest maturity and the highest coupon, it suffers the least change in price given a change in interest rates. Contrast that with bond D which has the lowest coupon and the highest time to maturity. It suffers the biggest hit in response to interest rate changes.

Bottom Line:
THE ZERO COUPON BOND SUFFERS THE BIGGEST PRICE SWING!

The further out until maturity the greater the price change. People often confuse the relationship based on coupon rate, just remember the zero coupon bond suffers the most.

Bond Pricing Relationships

- As maturity increases, price sensitivity increases but at a decreasing rate
- Price sensitivity is inversely related to a bond's coupon rate
- Price sensitivity is inversely related to the yield to maturity at which the bond is selling

This is saying:
THE HIGHER THE YIELD TO MATURITY THE LOWER THE PRICE SENSITIVITY.

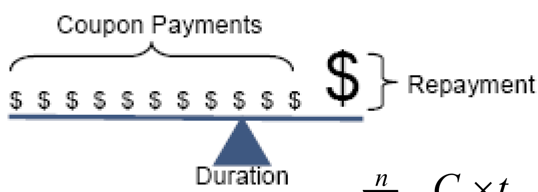
Extreme Examples with Equal Maturities

- Consider TWO TEN-YEAR MATURITY INSTRUMENTS:
  - A ten-year zero coupon bond
  - A two-cash flow "bond" that pays $999.99 almost immediately and one penny, ten years hence
- Small changes in yield to maturity will have a large effect on the value of the zero coupon bond but essentially no impact on the hypothetical bond
- Most bonds are between these extremes

Duration

- **WEIGHTED AVERAGE TIME TO MATURITY** using the relative present values of the cash flows as weights
- Combines the effects of differences in coupon rates and differences in maturity
- Based on elasticity of bond price with respect to interest rate
- **Duration is the point at which the cash flows balance out**
- In other words:

The point when payments already made to bondholders equal the payments yet to come.



Duration: $$D = \frac{\sum_{t=1}^{n} \left(\frac{C_t \times t}{(1+r)^t} \right)}{\sum_{t=1}^{n} \left(\frac{C_t}{(1+r)^t} \right)}$$ where $$\text{Bond Price} = \sum_{t=1}^{n} \left(\frac{C_t}{(1+r)^t} \right)$$

Where

D = duration

t = number of periods in the future

C_t = cash flow to be delivered in t periods

n= term-to-maturity

r = yield to maturity (**per period basis**).

$$w_t = \frac{\text{CF}_t/(1+y)^t}{\text{Bond price}} \qquad D = \sum_{t=1}^{T} t \times w_t$$

Bottom Line:
Duration will measure price sensitivity to interest rate changes.

Gives us the ability to say that if we experience a given shock in market interest rates of so many basis points this is the approximate change in the price of the bond.

Definition: weighted average time to maturity using relative present values. (a measure of bond price sensitivity)

The effects of coupons and time to maturity are both accounted for in a single number. This one value will give us an idea into the bonds sensitivity to interest rate changes.

The price of the bond is equal to the present value of the expected cash flows. What does this physically mean? Duration is simply the present value of all the cash flows multiplied by the time divided by the price.

$$\text{Price of Bond} = \sum_{t=1}^{n} \left(\frac{C_t}{(1+r)^t} \right)$$

What is the rate of change of the price of the bond given a change in market interest rates?

Normally with bonds we have $C_1 = C_2 = C_3 = \ldots = C_{t-1}$ and $C_T = F + C_1$

Now, what happens to the price of the bond for a change in market interest rate?

$$\frac{dP}{dr} = \frac{d}{dr}\sum_{t=1}^{T}\frac{C_t}{(1+r)^t} = \sum_{t=1}^{T}\frac{d}{dr}\frac{C_t}{(1+r)^t} = -\sum_{t=1}^{T}\frac{t \times C_t}{(1+r)^{t+1}} = -\frac{1}{(1+r)}\sum_{t=1}^{T}\frac{t \times C_t}{(1+r)^t}$$

$$\frac{dP}{dr} \approx \frac{\square}{\square} \qquad \text{and} \qquad \frac{\square}{P} \approx \frac{-\frac{1}{\quad}\sum_{t=1}^{T}\frac{t \times C_t}{(1+r)^t} \times \square}{P}$$

$$\text{so that} \qquad \text{Duration of a Bond} = D = \frac{\sum_{t=1}^{T}\frac{t \times C_t}{(1+r)^t}}{P} \qquad \text{where} \qquad \frac{\square}{P} \approx -\frac{) \times \square}{(1+r)}$$

Linear relationship between the change in the interest rate and the change in the price of the bond.

For a zero-coupon bond, because 100% of the value is contained in the final payment therefore we have:

since it is a zero-coupon: $C_1 = C_2 = C_3 = \ldots = C_{T-1} = 0$ (all coupon terms except one disappear)

for a zero-coupon bond: $D = \dfrac{T \times \dfrac{FaceValue}{(1+r)^T}}{\text{Price}}$ where $\text{Price} = \dfrac{\text{FaceValue}}{(1+r)^T}$

reduces to D=T for a zero-coupon bond!

(T is the time to maturity)

For all coupon paying bonds the DURATION is less than the time to maturity.

For a fixed maturity the zero-coupon bond will have the highest duration, every other bond has a duration less than a zero coupon bond.

For a given maturity the zero-coupon bond has the greatest price sensitivity.

Computing Duration

- Consider a 2-year, 8% coupon bond, with a face value of \$1,000 and yield-to-maturity of 12%. Coupons are paid semi-annually
- Therefore, each coupon payment is \$40 and the per period YTM is $\frac{12\%}{2} = 6\%$
- Present value of each cash flow equals $\frac{CF_t}{(1+0.06)^t}$ where t is the period number

2-year, 8% bond: Face value = \$1,000, YTM = 12%

t	years	CF_t	$PV(CF_t)$	Weight	Weight*Years
1	0.5	\$40	37.736	0.041	0.020
2	1	\$40	35.600	0.038	0.038
3	1.5	\$40	33.585	0.036	0.054
4	2	\$1,040	823.777	0.885	1.770
		Price=	930.698	D=	1.883
					years

note: weights must sum to 1.

$$w_t = \frac{CF_t/(1+y)^t}{\text{Bond price}}$$

$$D = \sum_{t=1}^{T} t \times w_t$$

ENSURE DURATION ANSWER < MATURITY !!!

Special Case

- Maturity of a **consol**: M = ∞
- Duration of a **consol**: D = 1 + 1/R where R is the YTM.

Console is a bond which pays interest in perpetuity, never repays the principle.

Features of Duration

- **DURATION AND MATURITY**:
  - ∗ D increases with M, but at a decreasing rate

    The longer the maturity the longer the duration.

- **DURATION AND YIELD-TO-MATURITY**:
  - ∗ D decreases as yield increases

    In two interest rate environments, one with low yields and one with high yields, a given interest rate shock is going to impact me more when my yields are low (when I am operating in the low yield environment). Lower yields and an interest rate shock will cause me more price fluctuation.

- **DURATION AND COUPON INTEREST**:
  - ∗ D decreases as coupon increases

    The bigger the coupon the lower the duration (get my money sooner).

Rules for Duration

Rule 1. The duration of a zero-coupon bond equals its time to maturity

Rule 2. Holding maturity constant, a bond's duration is higher when the coupon rate is lower

Rule 3. Holding the coupon rate constant, a bond's duration increases with its time to maturity

Rule 4. Holding other factors constant, the duration of a coupon bond is higher when the bond's yield to maturity is lower (takes longer to get my money) Lower ytm means high duration.

Economic Interpretation

- Duration is a measure of interest rate sensitivity or elasticity of a liability or asset:

[dP/P] ÷ [dR/(1+R)] = -D

$$\frac{\frac{dP}{P}}{\frac{\square}{(1+R)}} = -D$$

- Or equivalently,

dP/P = -D[dR/(1+R)] = -MD × dR

$$\frac{dP}{P} = -D \times \frac{\square}{(1+R)} = -MD \times \square \qquad \square \quad \frac{D}{(1+R)}$$

where ***MD*** is ***MODIFIED DURATION***

- To **estimate the change in price**, we can rewrite this as:

dP = -D[dR/(1+R)]P = -(MD) × (dR) × (P)

$$dP = -D \times \frac{\square}{(1+R)} \times P \approx -MD \times \square$$ ere $MD \square \frac{D}{(1+R)}$ $\square$ ate shock

Note the direct *linear* relationship between *dP and -D*

(This is what we have been after)

Can use this to measure a banks long portfolio (or anything else). P can refer to a portfolio of bonds, a banks balance sheet, a banks equity position. We are able to measure duration of combined assets and liabilities. Banks will use this to estimate the impact of interest rate changes.

EXAMPLE

Suppose I have a $1 million long position in a 7 year zero-coupon bond with annual YTM = 5%.

Q1. What is the value of the bond? (no coupon payments, only principle at end)

Q2. What is the approximate impact of a 1 basis point change in YTM on the value of my position?

A1.
$$\frac{\$1{,}000{,}000}{(1+.05)^7} = \$710{,}681$$

A2. $D \times P \times \quad \frac{D}{(1+R)} \quad \frac{7}{1.05} \quad 681 \times 0.0001 = \$473.79$

so DV01 = $473.79

to check $\frac{\$1{,}000{,}000}{(1+0.05+0.0001)^7} = \$710{,}207.72$ actual change= $473.28

adding in the 1 basis point change. This calculation gives us the true change.

DV01 means the impact on my position of a 1 basis point change in interest rates.

If interest rates go up, your value goes down so you don't have to put the negative sign on the DV01 value.

EXAMPLE

Calculate DV01 for previous coupon bond example (slide 43 reproduced below).

2-year, 8% bond: Face value = \$1,000, YTM = 12%

t	years	CF_t	$PV(CF_t)$	Weight	Weight*Years
1	0.5	\$40	37.736	0.041	0.020
2	1	\$40	35.600	0.038	0.038
3	1.5	\$40	33.585	0.036	0.054
4	2	\$1,040	823.777	0.885	1.770
		Price=	930.698	D=	1.883
					years

note: weights must sum to 1.

ENSURE DURATION ANSWER < MATURITY !!!

$$w_t = \frac{CF_t/(1+y)^t}{\text{Bond price}}$$

$$D = \sum_{t=1}^{T} t \times w_t$$

From the solution we have Price = \$930.689 and D = 1.883

$$D \times P \times \quad \frac{D}{(1+R)} \quad \frac{1.883}{1+0.12} \quad 930.698 \times 0.0001 = \$0.156$$

so DV01 = 15.6 cents for this bond

exact solution: $\frac{1.883}{(1+0.06)^2} \times \$930.698 \times 0.0001 = \$0.15597$

15 cents off on a 1 basis point shock

So a 1 basis point shock causes a 15 cent loss on a \$1000 bond.

Examples (slide we did not do)

- T-Bill
- 30 Year T-Bond

SUMMARY

1. Fixed-income securities are distinguished by their promise to pay a fixed or specified stream of income to their holders. The coupon bond is a typical fixed-income security.

2. Treasury notes and bonds have original maturities greater than one year. They are issued at or near par value, with their prices quoted net of accrued interest. T-bonds may be callable during their last five years of life.

3. Callable bonds should offer higher promised yields to maturity to compensate investors for the fact that they will not realize full capital gains should the interest rate fall and the bonds be called away from them at the stipulated call price. Bonds often are issued with a period of call protection. In addition, discount bonds selling significantly below their call price offer implicit call protection.

4. Put bonds give the bondholder rather than the issuer the option to terminate or extend the life of the bond.

5. Convertible bonds may be exchanged, at the bondholder's discretion, for a specified number of shares of stock. Convertible bondholders "pay" for this option by accepting a lower coupon rate on the security.

6. Floating-rate bonds pay a coupon rate at a fixed premium over a reference short-term interest rate. Risk is limited because the rate is tied to current market conditions.

7. The yield to maturity is the single interest rate that equates the present value of a security's cash flows to its price. Bond prices and yields are inversely related. For premium bonds, the coupon rate is greater than the current yield, which is greater than the yield to maturity. The order of these inequalities is reversed for discount bonds.

8. The yield to maturity is often interpreted as an estimate of the average rate of return to an investor who purchases a bond and holds it until maturity. This interpretation is subject to error, however. Related measures are yield to call, realized compound yield, and expected (versus promised) yield to maturity.

9. Prices of zero-coupon bonds rise exponentially over time, providing a rate of appreciation equal to the interest rate. The IRS treats this price appreciation as imputed taxable interest income to the investor.

10. When bonds are subject to potential default, the stated yield to maturity is the maximum possible yield to maturity that can be realized by the bondholder. In the event of default, however, that promised yield will not be realized. To compensate bond investors for default risk, bonds must offer default premiums, that is, promised yields in excess of those offered by default-free government securities. If the firm remains healthy, its bonds will provide higher returns than government bonds. Otherwise the returns may be lower.

11. Bond safety is often measured using financial ratio analysis. Bond indentures are another safeguard to protect the claims of bondholders. Common indentures specify sinking fund requirements, collateralization

Text Chapter 14.